

# NLP1

## Syntax (and Morphology)

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# Where are we at?

## Week 1

- HC1a: text classification
- HC1b: language modelling

## Week 2

- HC2a: sequence labelling
- Today: **syntax (and morphology)**

# Modelling Language Thus Far

Bag-of-words (e.g., NB)

- ignore word order entirely



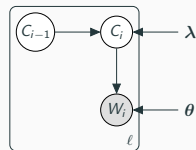
Markov models (e.g., bigram LM)

- memorise short observed phrases



HMM

- capture shallow syntactic patterns through adjacent word classes
- no semantic dependency amongst words



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If you want to learn more about how to read these diagrams, you can check a link from our earlier classes: [introduction to PGMs](#)

Much like we abstracted from words to their (syntactic) categories, we can abstract from phrases to their syntactic categories.



# Generalising POS categories

POS categories indicate which words are substitutable:

I saw a [ADJ] cat.

Phrasal categories indicate which *phrases* are substitutable:

[NP. . .] sleep soundly.

Phrasal categories; noun phrase (NP), verb phrase (VP), prepositional phrase (PP), etc.

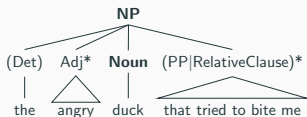
# Heads and Phrases

The class that a word belongs to is closely linked to the name of the phrase it customarily appears in.

In English,

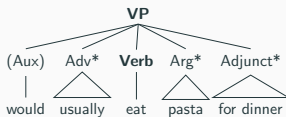
NPs are commonly of the form

- (Det) Adj\* **Noun** (PP|RelClause)\*



VPs are commonly of the form

- (Aux) Adv\* **Verb** Arg\* Adjunct\*



# Constituency

Syntactic constituency is the idea that groups of words can behave as single units, or constituents.

Example: noun phrases (NPs)

Harry the Horse  
the Broadway coppers  
they

a high-class spot such as Mindy's  
the reason he comes into the Hot Box  
three parties from Brooklyn

One evidence for their existence is that they appear in similar syntactic environments (e.g., NPs tend to appear before a verb).

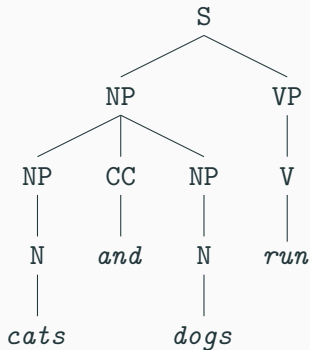
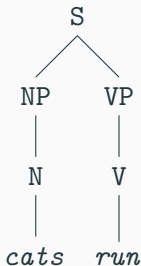
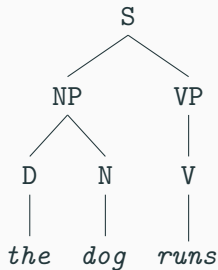
three parties from Brooklyn *arrive...*  
a high-class spot such as Mindy's *attracts...*  
the Broadway coppers *love...*  
they *sit*



# Nesting

Constituents can be hierarchically embedded in other constituents.

You can view the result as a tree-like structure.



# Context-Free Grammars

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# Context-Free Grammar (CFG)

A rewriting system with two types of *symbols* and a set of *symbol-rewriting rules*.

## Symbols

- *Terminals* (or *constants*): words;
- *Nonterminals* (or *variables*): word and phrasal categories.

## Rules

- $X \rightarrow \beta$  where  $X$  is a nonterminal, and  $\beta$  is any string of terminal and nonterminal symbols.

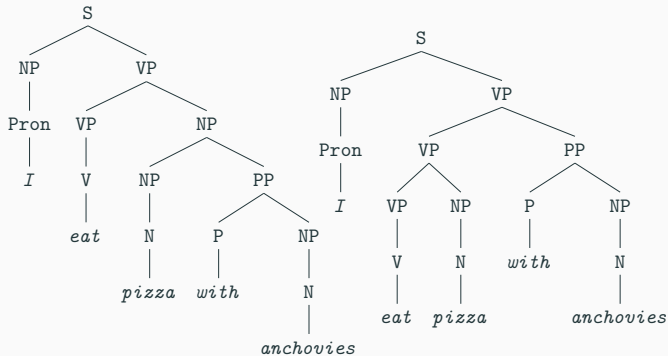
# Example

Nonterminals: S, NP, VP, PP, Pron, N, V, P

Terminals: *I, eat, pizza, with, anchovies*

- $S \rightarrow NP VP$
- $NP \rightarrow NP PP$
- $VP \rightarrow VP PP$
- $VP \rightarrow VP NP$
- $PP \rightarrow P NP$
- $VP \rightarrow V$
- $NP \rightarrow Pron$
- $NP \rightarrow N$
- $V \rightarrow eat$
- $Pron \rightarrow I$
- $P \rightarrow with$
- $N \rightarrow pizza$
- $N \rightarrow anchovies$

Text: *I eat pizza with anchovies*



$\Sigma$  is a finite set of terminal symbols

$\mathcal{V}$  is a finite set of nonterminal symbols, with a distinguished start symbol  $S \in \mathcal{V}$  and  $\mathcal{V} \cap \Sigma = \emptyset$

$\mathcal{R}$  is a finite set of rules of the kind:  $X \rightarrow \beta$  with  $X \in \mathcal{V}$  and  $\beta \in (\Sigma \cup \mathcal{V})^*$ .

A CFG is the tuple  $\langle \Sigma, \mathcal{V}, S, \mathcal{R} \rangle$

Arity (length of rule's RHS)

- unary:  $A \rightarrow B$
- binary:  $X \rightarrow BC$
- $n$ -ary:  $X \rightarrow X_1, \dots, X_n$
- if the longest rule has arity  $a$ , we say the grammar has arity  $a$

No matter the CFG, we can always re-express the same set of strings in *Chomsky normal form* (CNF), which gives us a grammar of arity 2.

We can use CFGs to **derive** strings

A **derivation** is a sequence of strings

- we start from the string  $\langle S \rangle$
- and at each step we rewrite the **leftmost** nonterminal  $X$  by application of a rule  $X \rightarrow \beta$
- until only terminals remain

If a string  $w_1 \cdots w_\ell$  is derivable from  $S$  we write:  $S \xRightarrow{*} w_1 \cdots w_\ell$ .



## Example

Rule	Derivation
$\text{BoS} \rightarrow S$	$\langle S \rangle$

## Example

Rule	Derivation
$\text{BoS} \rightarrow \text{S}$	$\langle \text{S} \rangle$
$\text{S} \rightarrow \text{NP VP}$	$\langle \text{NP VP} \rangle$

## Example

Rule	Derivation
$\text{BoS} \rightarrow \text{S}$	$\langle \text{S} \rangle$
$\text{S} \rightarrow \text{NP VP}$	$\langle \text{NP VP} \rangle$
$\text{NP} \rightarrow \text{D N}$	$\langle \text{D N VP} \rangle$

## Example

Rule	Derivation
$\text{BoS} \rightarrow \text{S}$	$\langle \text{S} \rangle$
$\text{S} \rightarrow \text{NP VP}$	$\langle \text{NP VP} \rangle$
$\text{NP} \rightarrow \text{D N}$	$\langle \text{D N VP} \rangle$
$\text{D} \rightarrow \textit{the}$	$\langle \textit{the} \text{ N VP} \rangle$

## Example

Rule	Derivation
$\text{BoS} \rightarrow \text{S}$	$\langle \text{S} \rangle$
$\text{S} \rightarrow \text{NP VP}$	$\langle \text{NP VP} \rangle$
$\text{NP} \rightarrow \text{D N}$	$\langle \text{D N VP} \rangle$
$\text{D} \rightarrow \textit{the}$	$\langle \textit{the} \text{ N VP} \rangle$
$\text{N} \rightarrow \textit{dog}$	$\langle \textit{the dog VP} \rangle$

## Example

Rule	Derivation
$\text{BoS} \rightarrow \text{S}$	$\langle \text{S} \rangle$
$\text{S} \rightarrow \text{NP VP}$	$\langle \text{NP VP} \rangle$
$\text{NP} \rightarrow \text{D N}$	$\langle \text{D N VP} \rangle$
$\text{D} \rightarrow \textit{the}$	$\langle \textit{the} \text{ N VP} \rangle$
$\text{N} \rightarrow \textit{dog}$	$\langle \textit{the} \textit{dog} \text{ VP} \rangle$
$\text{VP} \rightarrow \text{V}$	$\langle \textit{the} \textit{dog} \text{ V} \rangle$

## Example

Rule	Derivation
$\text{BoS} \rightarrow \text{S}$	$\langle \text{S} \rangle$
$\text{S} \rightarrow \text{NP VP}$	$\langle \text{NP VP} \rangle$
$\text{NP} \rightarrow \text{D N}$	$\langle \text{D N VP} \rangle$
$\text{D} \rightarrow \textit{the}$	$\langle \textit{the} \text{ N VP} \rangle$
$\text{N} \rightarrow \textit{dog}$	$\langle \textit{the dog VP} \rangle$
$\text{VP} \rightarrow \text{V}$	$\langle \textit{the dog V} \rangle$
$\text{V} \rightarrow \textit{barks}$	$\langle \textit{the dog barks} \rangle$

We can denote the derivation (i.e., sequence of rule applications) by  $\delta$ .

The fact that  $\delta$  derives a specific string (e.g., *the dog barks*) can be denoted by  $\text{S} \xRightarrow{\delta} w_1 \cdots w_\ell$ .

## So we write some rules. What could go wrong?

On Mentimeter...



# Probabilistic Context-Free Grammars

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## Probabilistic CFG (PCFG)

A probability distribution over the space of all derivations (including their yields) supported by a grammar.

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Dyer et al. [2016] introduce a neural parameterisation of such as model.

# Factorisation

A random derivation  $D = \langle R_1, \dots, R_M \rangle$  is a sequence of  $M$  random rule applications. A valid derivation rewrites  $S$  into a sequence of random words  $X = \langle W_1, \dots, W_L \rangle$ .

The CFG backbone gives us a mechanism to factorise probabilities. We can assign probability mass to  $r_{1:m}$  via chain rule

$$P_D(r_{1:m}) = \prod_{j=1}^m P_{R|H}(r_j | r_{<j})$$

$r_{<j}$  is the *history* of rule applications relative to the  $j$ th rule in the derivation.

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Dyer et al. [2016] introduce a neural parameterisation of such a model.

# Markov Model over Steps

A random derivation  $D = \langle R_1, \dots, R_M \rangle$  is a sequence of  $M$  random rule applications.

A random rule is a pair  $(N, S)$  of a random LHS nonterminal and a random RHS string.

We can assign probability mass to  $r_{1:m}$  via chain rule under a Markov assumption:

$$P_D(r_{1:m}) \stackrel{\text{ind.}}{=} \prod_{j=1}^m P_R(r_j) = \prod_{j=1}^m P_{S|N}(\beta_j | v_j)$$

with  $v_j \in \mathcal{V}$  and  $\beta_j \in (\mathcal{V} \cup \Sigma)^*$

Note: pretend every derivation starts with  $r_0 = \text{BoS} \rightarrow S$ .

## Example

Rule Probability	Derivation
1	$\langle S \rangle$
$P_{S N}(\text{NP VP}   S)$	$\langle \text{NP VP} \rangle$
$P_{S N}(\text{D N}   \text{NP})$	$\langle \text{D N VP} \rangle$
$P_{S N}(\text{the}   \text{D})$	$\langle \text{the N VP} \rangle$
$P_{S N}(\text{dog}   \text{N})$	$\langle \text{the dog VP} \rangle$
$P_{S N}(\text{V}   \text{VP})$	$\langle \text{the dog V} \rangle$
$P_{S N}(\text{barks}   \text{V})$	$\langle \text{the dog barks} \rangle$

1. Start with  $D = \langle S \rangle$
2. If all symbols in  $D$  are terminal, stop. Else, go to (3).
3. Condition on the left-most nonterminal symbol  $\nu$  in the derivation, and draw a RHS string  $\beta$  with probability  $P_{S|N}(\beta|\nu)$ , replace  $\nu$  in  $D$  by  $\beta$ . Repeat from (2).

This corresponds to a **depth-first** expansion of nonterminals. See my commented [Colab demo](#).

If we can rewrite a nonterminal variable  $\nu$  into  $K$  different ways, associate a  $K$ -dimensional Categorical distribution with  $S|N = \nu$ :

Examples:

$$S|N = \text{S} \sim \text{Categorical}(\theta_{\text{S} \rightarrow \text{NP VP}}, \theta_{\text{S} \rightarrow \text{VP}})$$

$$S|N = \text{N} \sim \text{Categorical}(\theta_{\text{N} \rightarrow \text{cat}}, \theta_{\text{N} \rightarrow \text{dog}}, \theta_{\text{N} \rightarrow \text{bird}})$$

The pmf assigns mass  $\prod_{j=1}^m \theta_{\nu_j \rightarrow \beta_j}$  to a derivation

$$r_{1:m} = \langle \nu_1 \rightarrow \beta_1, \dots, \nu_m \rightarrow \beta_m \rangle$$



Relative frequency of observed rule application

$$\theta_{\nu \rightarrow \beta} = \frac{\text{count}(\nu \rightarrow \beta)}{\sum_{(\nu \rightarrow \gamma) \in \mathcal{R}} \text{count}(\nu \rightarrow \gamma)}$$

# Evaluation

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Due to structural ambiguities, there are potentially many derivations for any one sentence  $w_{1:\ell}$ .

The PCFG assigns **marginal probability**

$$P_X(w_{1:\ell}) = \sum_{\delta: S \xRightarrow{\delta} w_{1:\ell}} P_D(\delta)$$

equal to the sum of the probabilities of **all derivations whose yield is the sentence we want a marginal probability for**.

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The sum is over the space of all derivations that have  $w_{1:\ell}$  as yield.

Assess perplexity of model using marginal probability of sentences in heldout dataset of valid sentences.

Obtain the **most probable derivation** subject to its yield being the sentence we want to parse:

$$\hat{\delta} = \arg \max_{\delta: S \xRightarrow{\delta} w_{1:\ell}} P_D(\delta)$$

Compare model prediction to a human annotated tree. Think of it in terms of span classification (i.e., did we classify  $w_{i:j}$  correctly as an NP?), report the constituent label precision, recall,  $F_1$ .

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See section 18.8 of textbook.

The key to both uses of PCFG (as an LM or as a parser) is to find all derivations of a given sentence  $w_{1:\ell}$ , a set we refer to as a **parse forest** for  $w_{1:\ell}$ .

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*Enumeration is bad.* We typically work with binary-branching trees (arity=2), then the number of trees for a sentence of  $L$  words is the **Catalan number**  $C_L = \frac{(2L)!}{(L+1)!L!}$ .

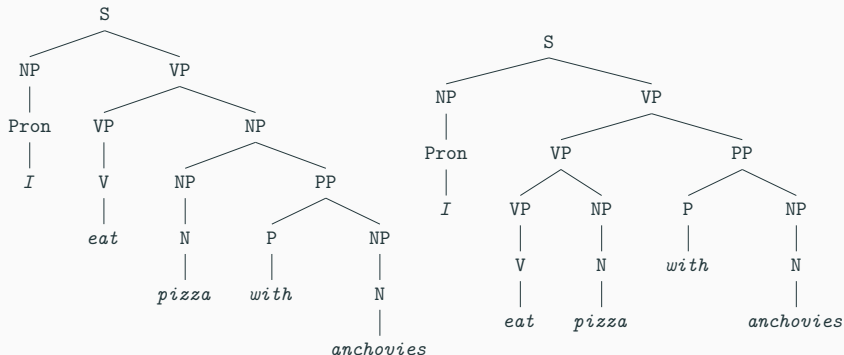


But to find the sum of probabilities or the maximum probability we do not need to **enumerate** the trees, we can exploit the Markov assumption in yet another dynamic programme.

**The CKY algorithm** is a compact representation of a forest. It can be used to find marginal probability (Inside algorithm) and maximum probability (Viterbi algorithm).

# CKY - Intuition

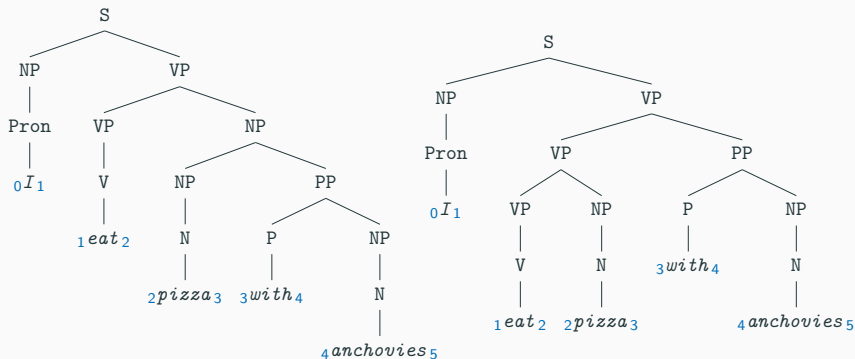
A *forest* is made of elementary building blocks that are reused:  
*phrase categories over input spans.*



# CKY - Intuition

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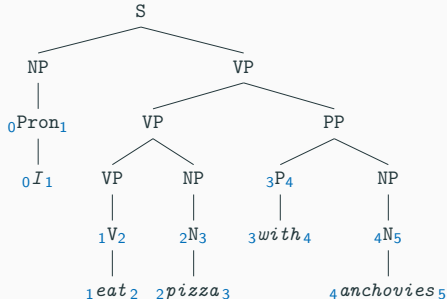
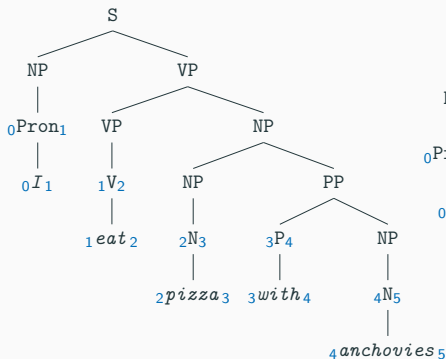
A span is identified by a pair of positions from 0 to  $\ell$



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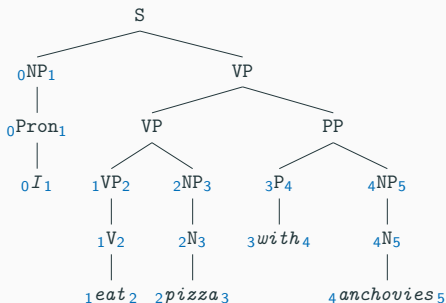
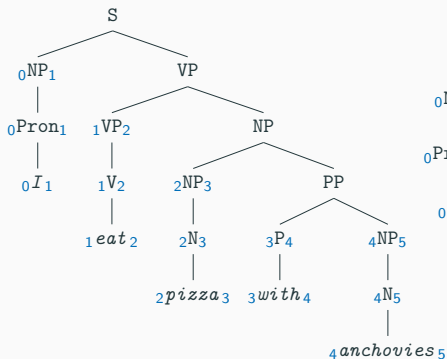
POS tags span single words



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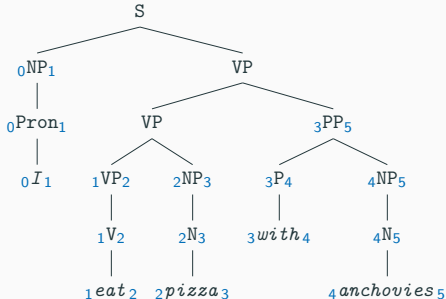
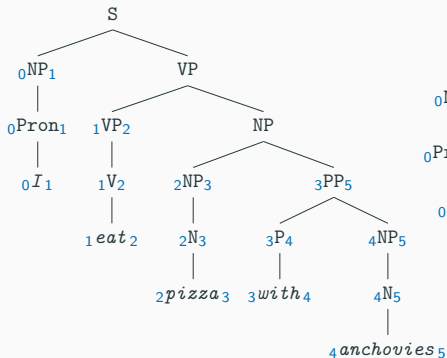
Some phrase categories are derived by unary rules



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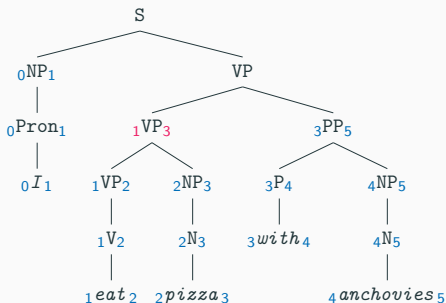
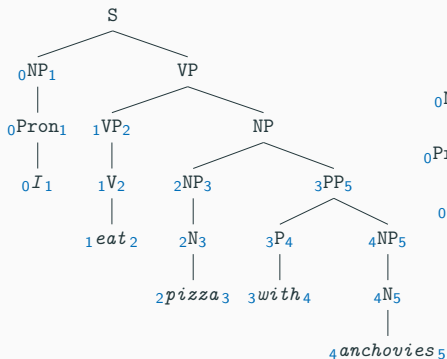
Some phrase categories merge two spans



# CKY - Intuition

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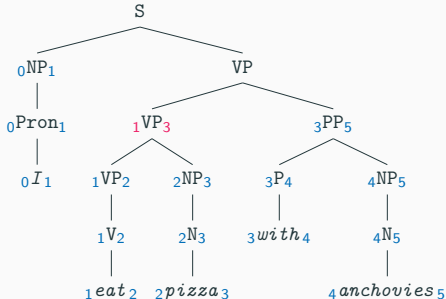
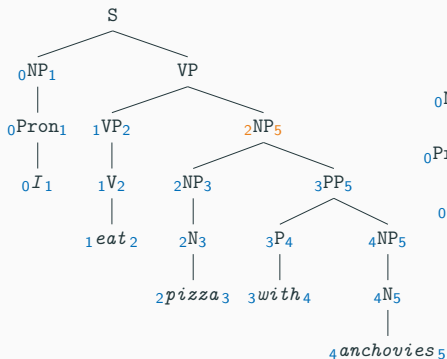
Some spans are common across derivations, while others



# CKY - Intuition

A *forest* is made of elementary building blocks that are reused:  
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Some spans are common across derivations, while others aren't

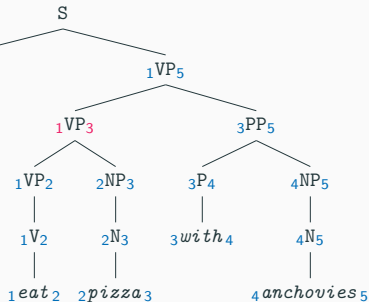
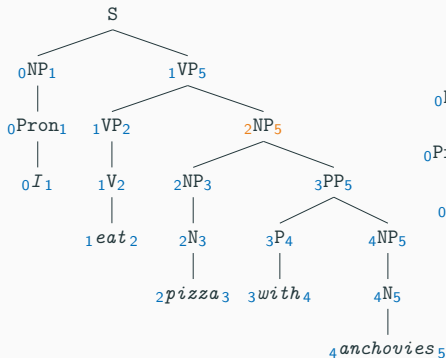




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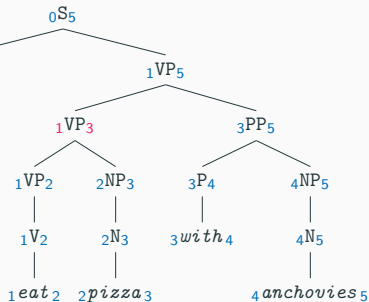
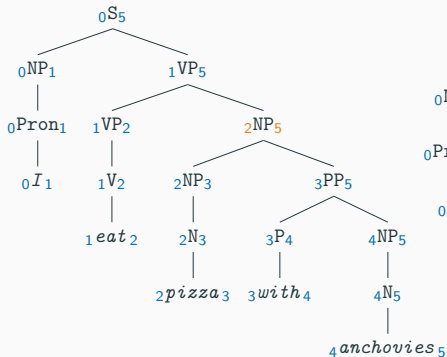
We may derive the same phrase category in different ways



# CKY - Intuition

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We may derive the same phrase category in different ways



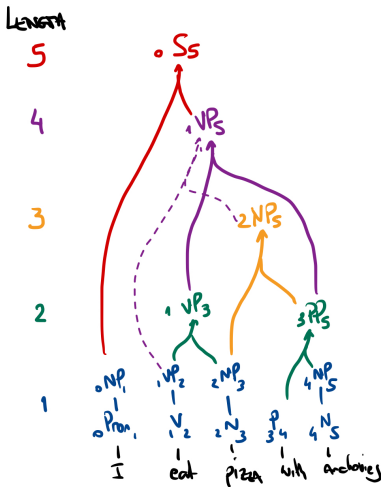
# CKY - Intuition

We can construct a graph-like view of the forest, compressed to size that is cubic in sentence length.

Resources:

- [my video tutorial](#)<sup>a</sup>
- [Appendix C](#) of textbook can be useful.

<sup>a</sup>On Canvas Media Gallery



Limited use of linguistic context due to generative formulation.

The context-free assumption is not enough in general: some linguistic constructions violate it.

Dynamic programming for PCFGs takes time that is cubic in sentence length.

Like the HMM, the PCFG is a generative model. If all we care about is a mechanism to predict parse trees, then we can parameterise the model conditionally using an expressive feature function. Examples: transition-based parsers, CRF parsers.

We may care about relations between words, more so than constituency, for that we develop **dependency grammars**.

Optional reading: [Chapter 19 of textbook](#).

## Self-study

*Videos on Canvas Media Gallery*

- Watch the video on [dynamic programming for PCFGs](#)
- Watch [Katia's class on Morphology](#)  
(first 43 minutes of the video)

Other, useful (but optional) material

- Check the colab demo on [sampling from PCFGs](#)
- [Chapter 18 of textbook](#)
- [Appendix C of textbook](#)

## References

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Chris Dyer, Adhiguna Kuncoro, Miguel Ballesteros, and Noah A. Smith. Recurrent neural network grammars. In Kevin Knight, Ani Nenkova, and Owen Rambow, editors, *Proceedings of the 2016 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pages 199–209, San Diego, California, June 2016. Association for Computational Linguistics. doi: 10.18653/v1/N16-1024. URL <https://aclanthology.org/N16-1024/>.