Natural Language Models and Interfaces

BSc Artificial Intelligence

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Institute for Logic, Language, and Computation

2019, week 5, lecture b

Context-Free Grammars

A **CFG** grammar G is denoted by

- a finite set of **nonterminal** symbols $\mathscr{V}$
- a finite set of **terminal** symbols Σ with $\Sigma \cap \mathscr{V} = \varnothing$
- a finite set $\mathscr{R}$ of **rules** of the form $X \rightarrow \beta$ where
  - $X \in \mathscr{V}$ and $\beta \in (\Sigma \cup \mathscr{V})^*$
- $S \in \mathscr{V}$ a distinguished **start** symbol

Let ε denote an **empty** string

Example CFG

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Generative Device

Left-most derivation

- sequence of strings $\alpha_1 \ldots \alpha_n$
  - $\alpha_1 = \langle S \rangle$
  - $\alpha_n \in \Sigma^*$
  - $\alpha_{i \geq 2}$ derived from α_{i-1} by picking the left-most nonterminal X
    - and replacing it by some α such that $X \to \beta \in \mathscr{R}$

Example of Derivation

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String	Substitution

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α_1 =	S	S → NP VP

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	String	Substitution
α_1 =	S	S → NP VP
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α_4 =	the NN VP	NN → man

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	String	Substitution
α_1 =	S	S → NP VP
α_2 =	NP VP	NP → DT NN
α_3 =	DT NN VP	DT → the
α_4 =	the NN VP	NN → man
α_5 =	the man VP	VP → Vi

Example of Derivation

	String	Substitution
α_1 =	S	S → NP VP
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α_3 =	DT NN VP	DT → the
α_4 =	the NN VP	NN → man
α_5 =	the man VP	VP → Vi
α_6 =	the man Vi	Vi → sleeps

Example of Derivation

	String	Substitution
α_1 =	S	S → NP VP
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α_3 =	DT NN VP	DT → the
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α_6 =	the man Vi	Vi → sleeps
α_7 =	the man sleeps	

Example of Derivation

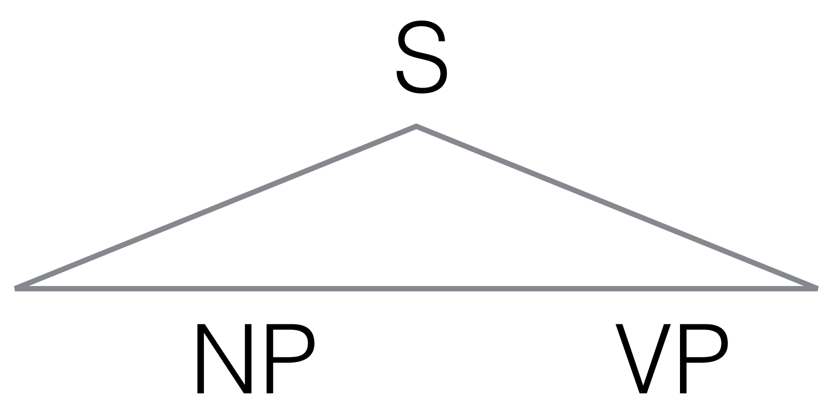
	String	Substitution
α_1 =	S	S → NP VP
α_2 =	NP VP	NP → DT NN
α_3 =	DT NN VP	DT → the
α_4 =	the NN VP	NN → man
α_5 =	the man VP	VP → Vi
α_6 =	the man Vi	Vi → sleeps
α_7 =	the man sleeps	
α_7 =	$S \Rightarrow^*$ the man sleeps	

Example of Generation

Example of Generation

S

Example of Generation



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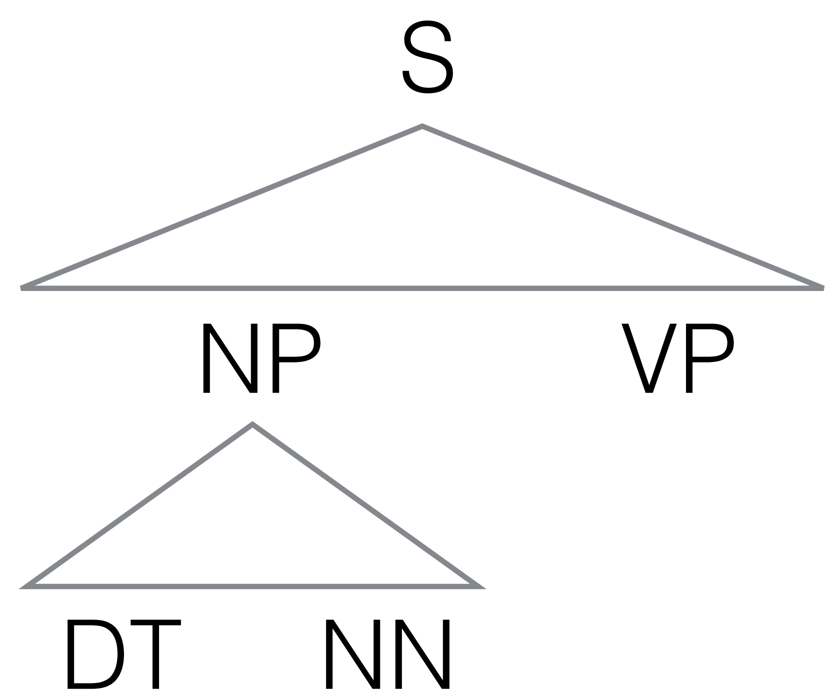

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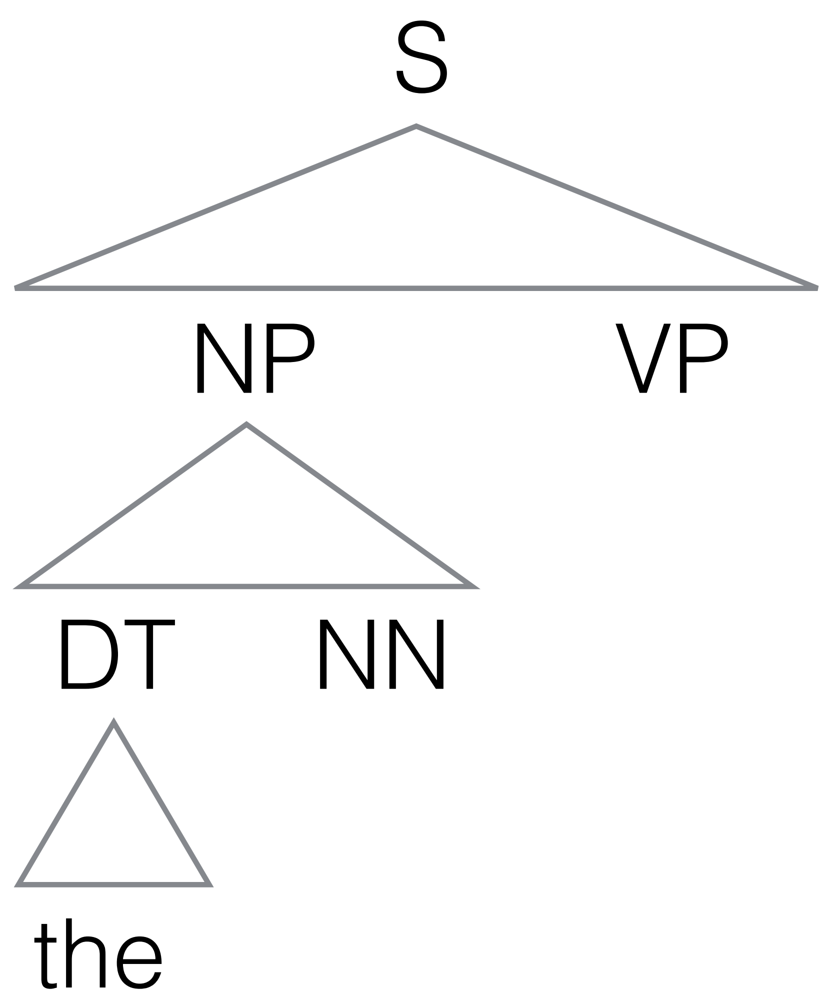

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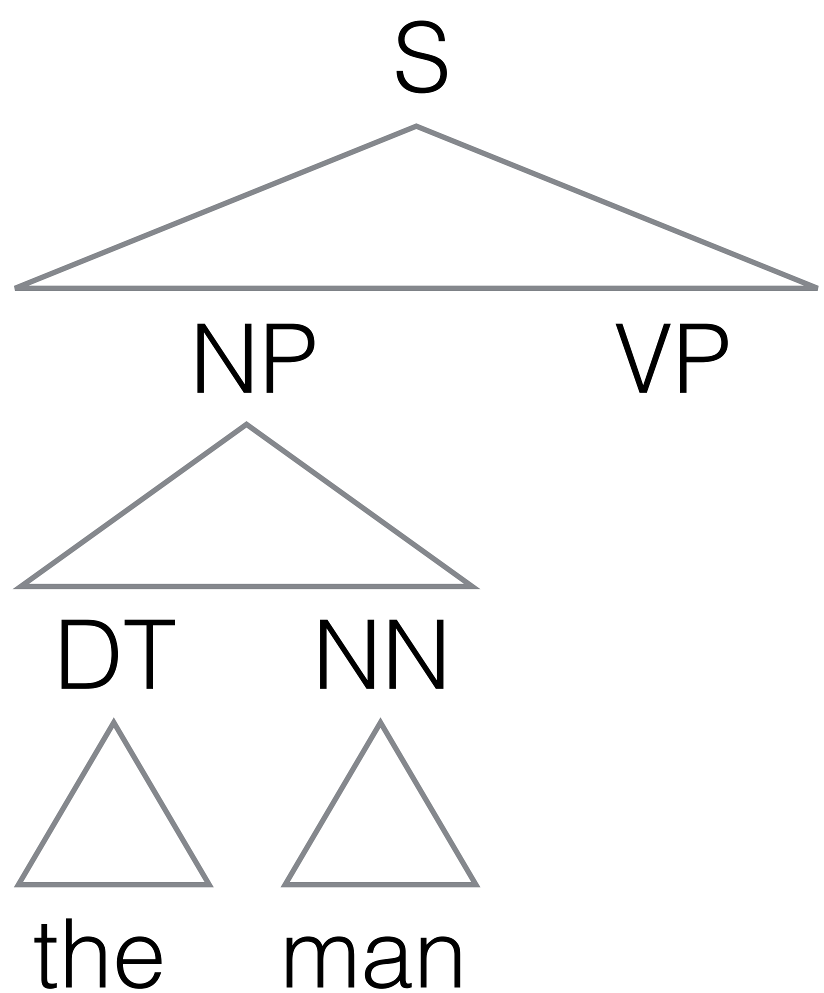

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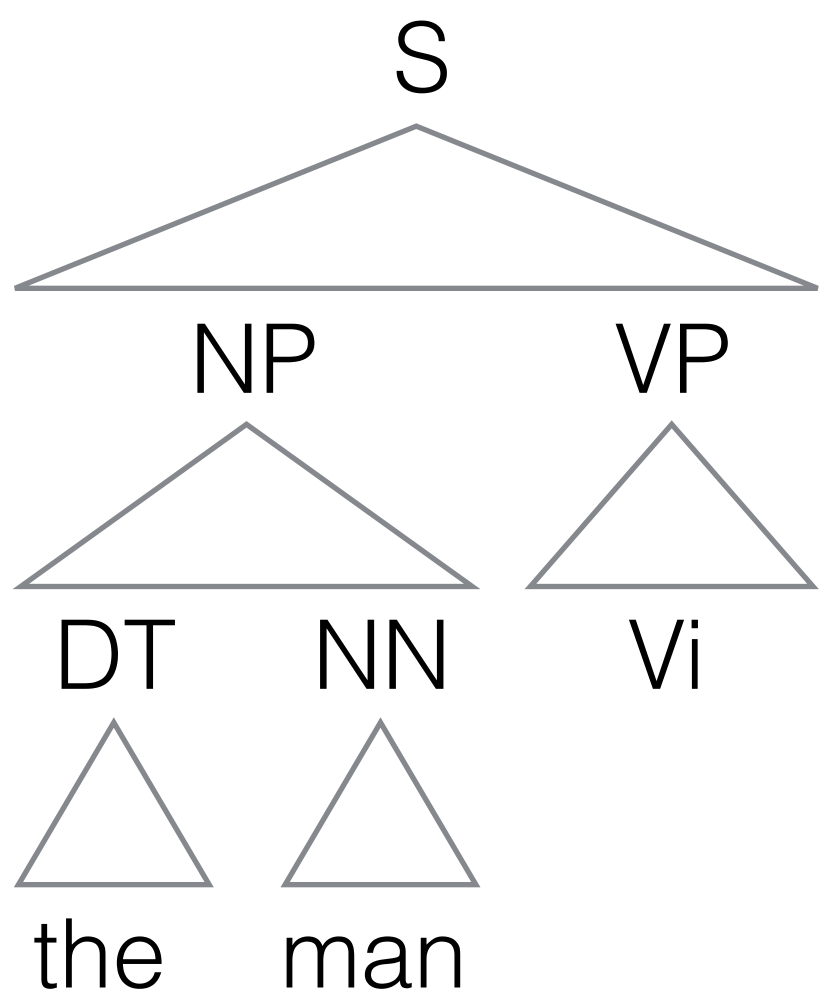

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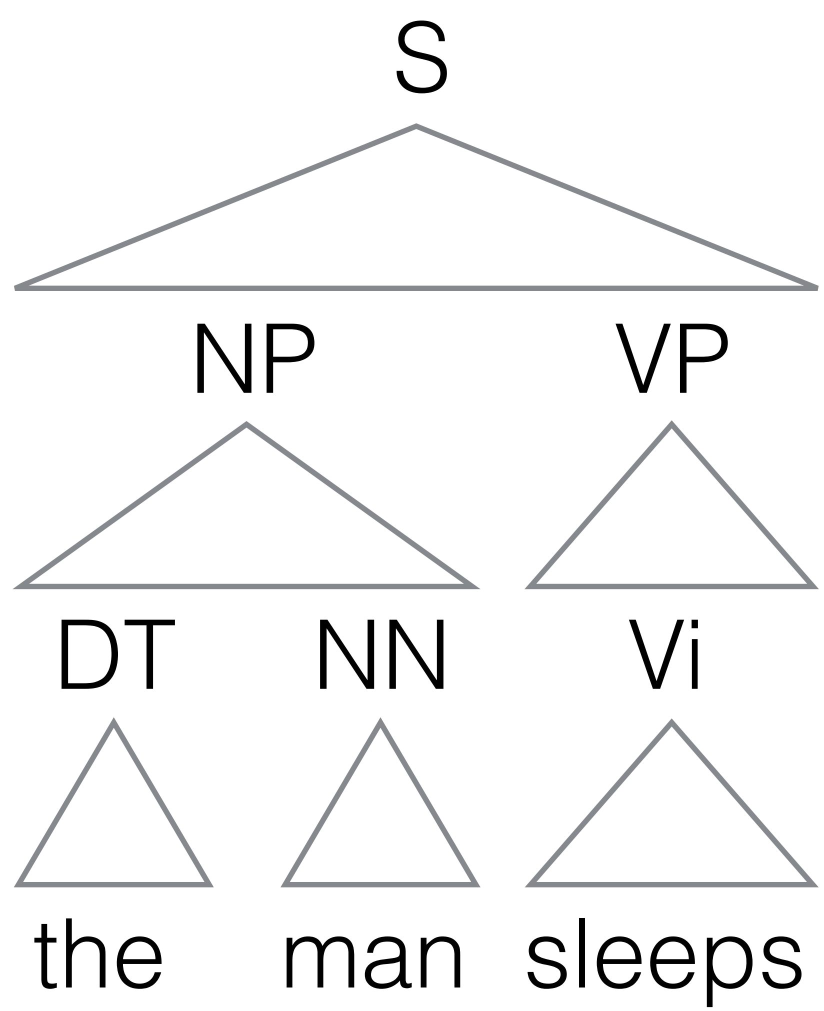

Example of Recognition

Example of Recognition

The man saw the dog

Example of Recognition

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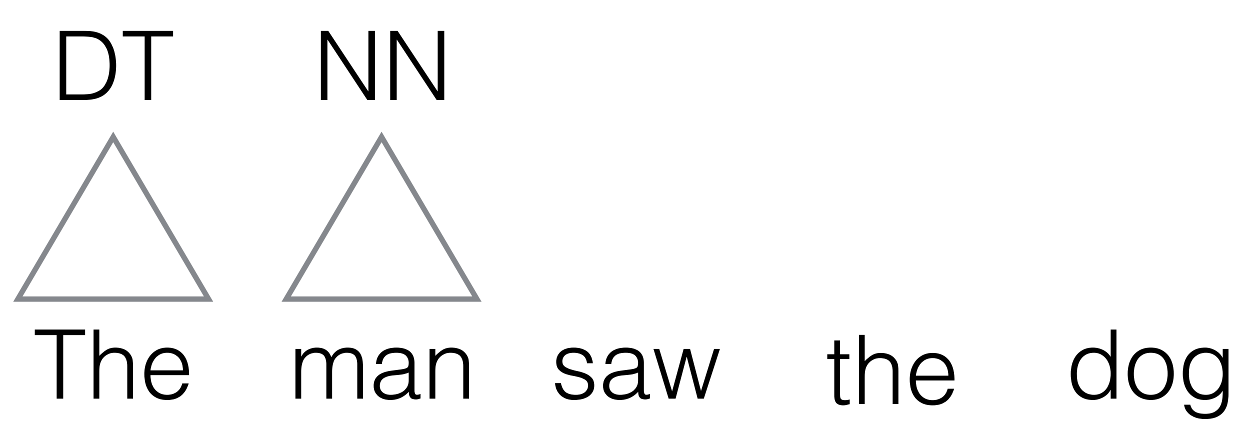

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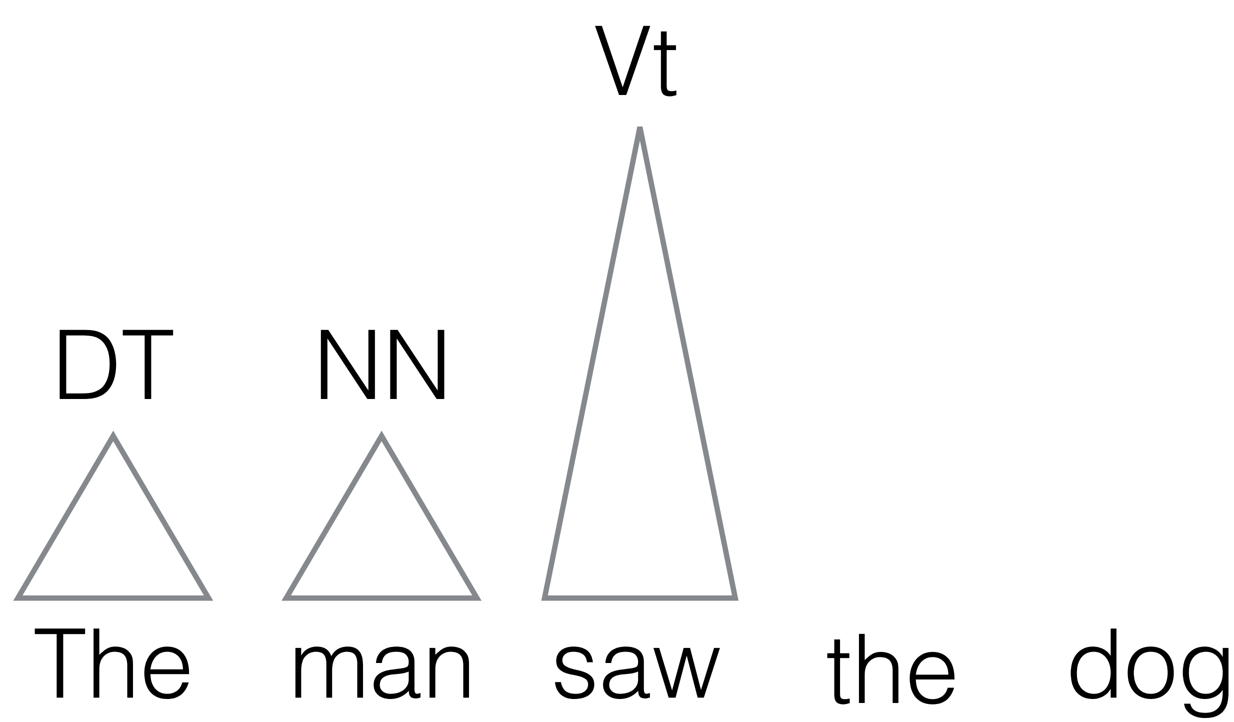

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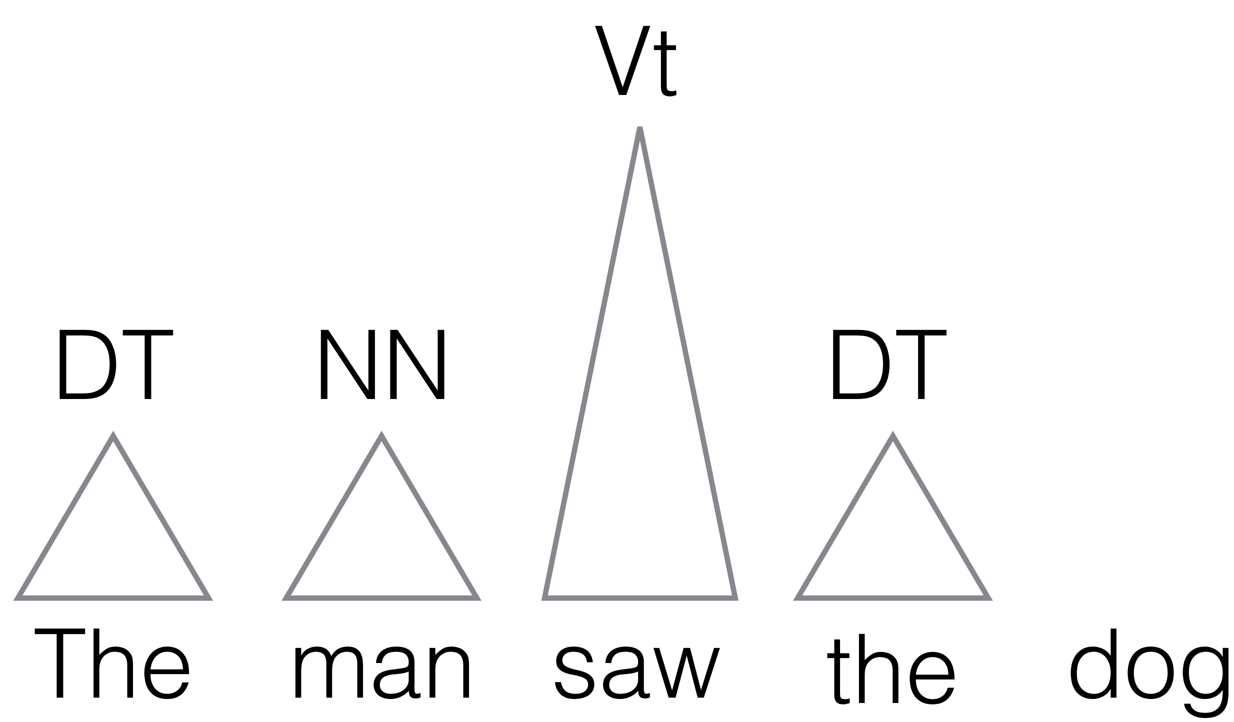

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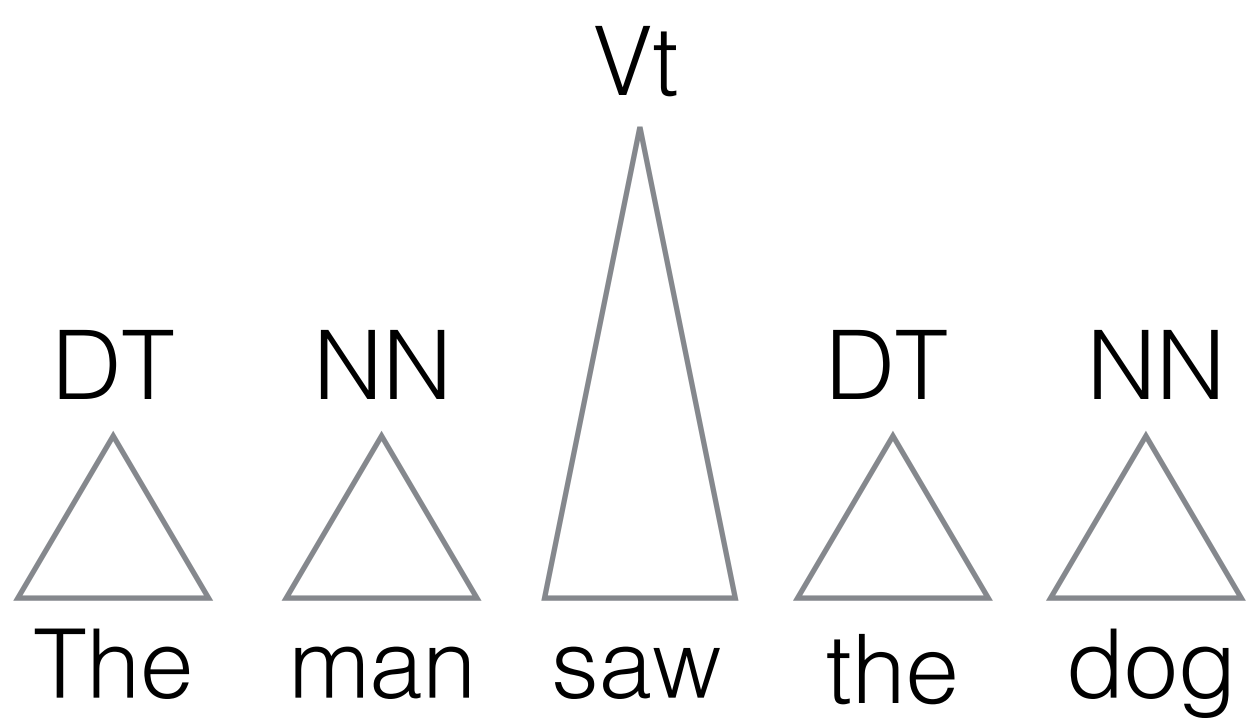

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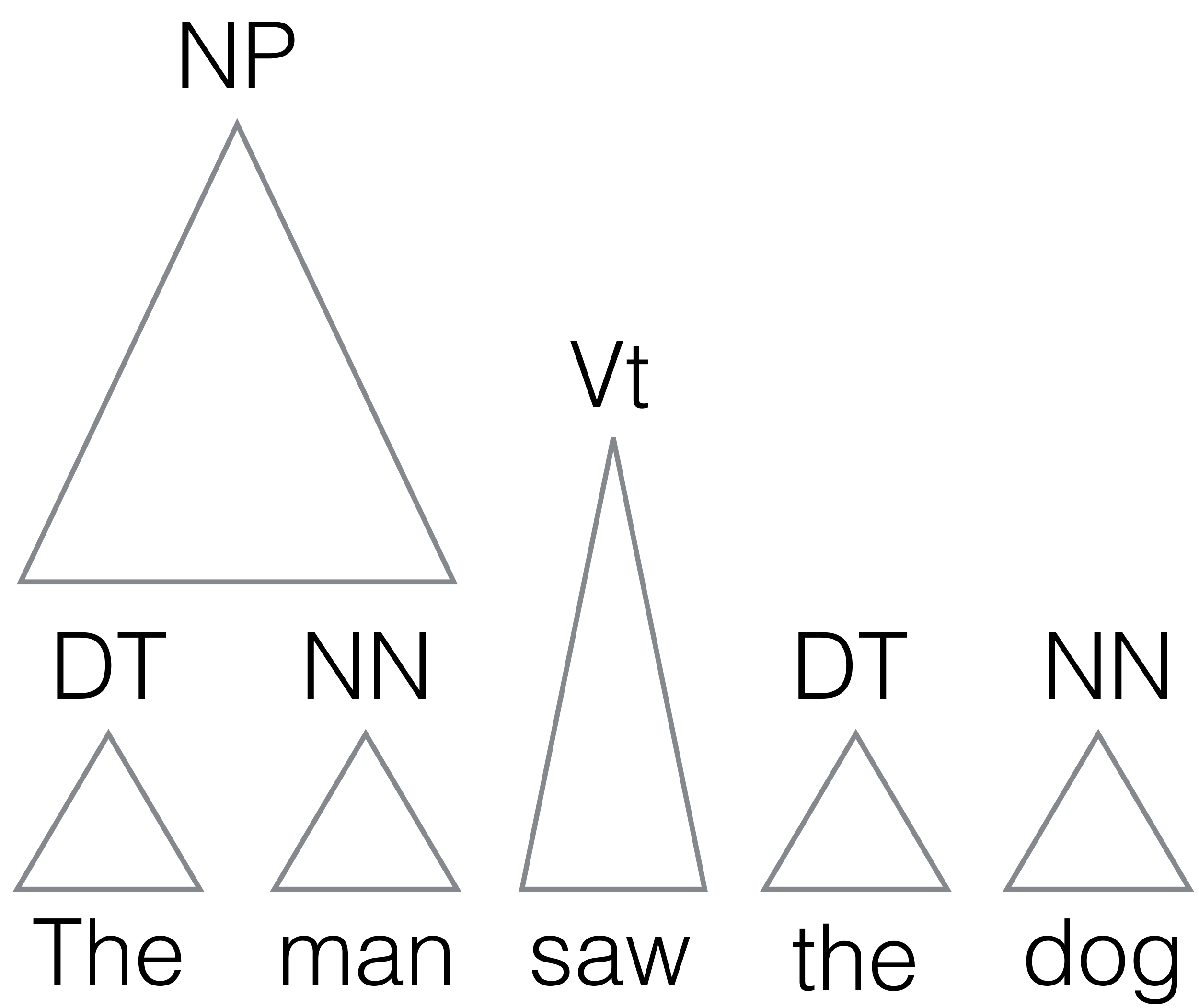

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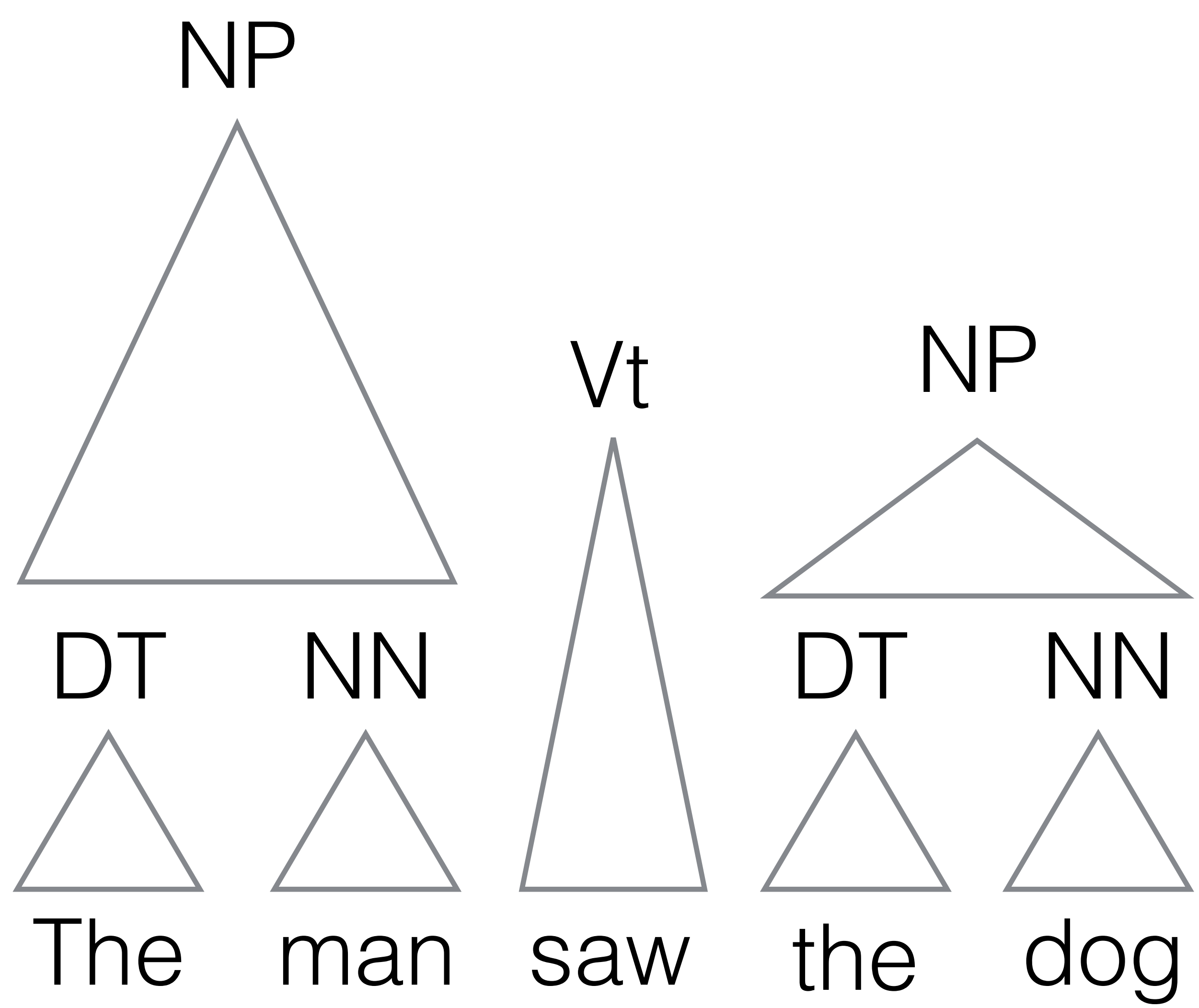

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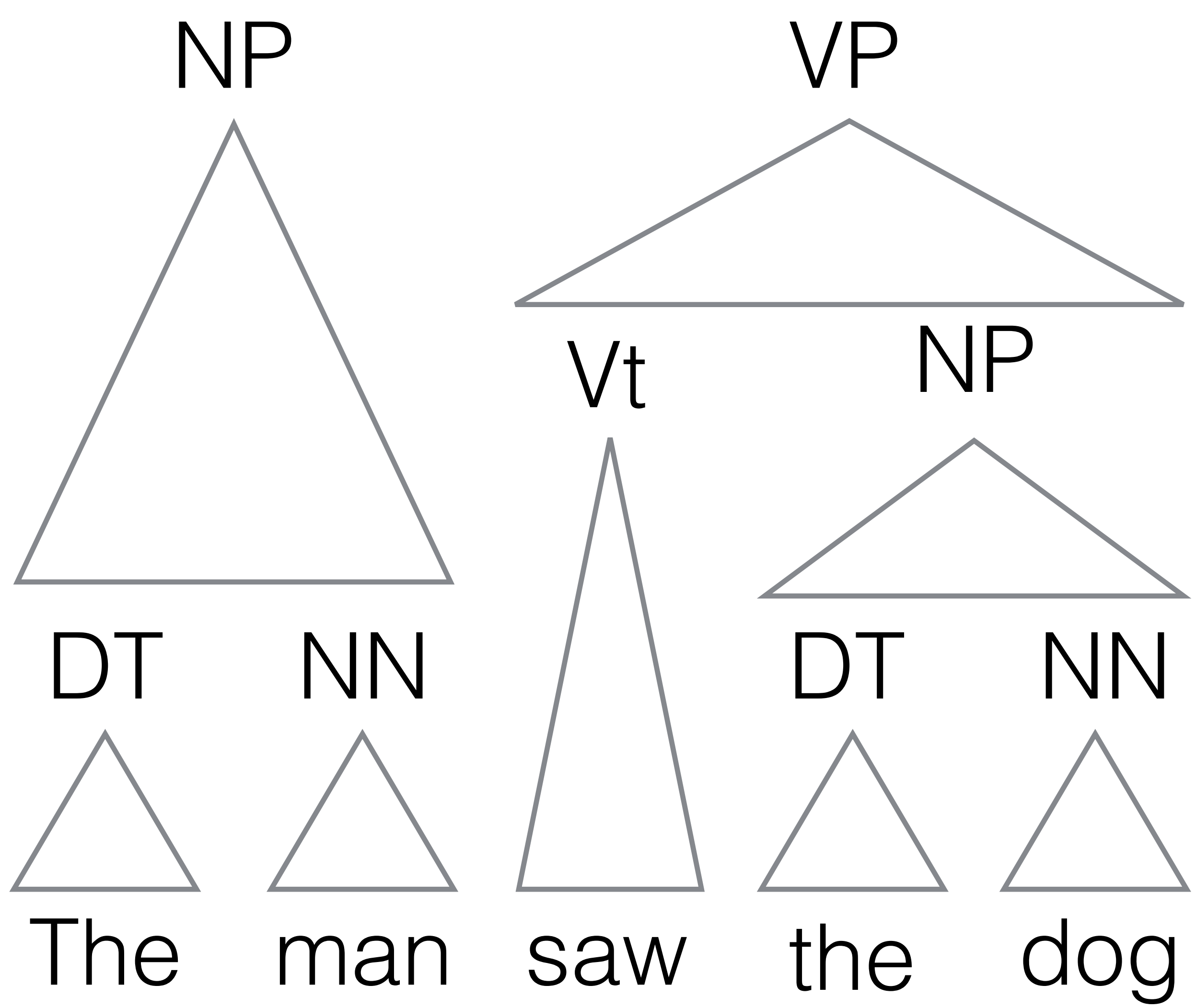

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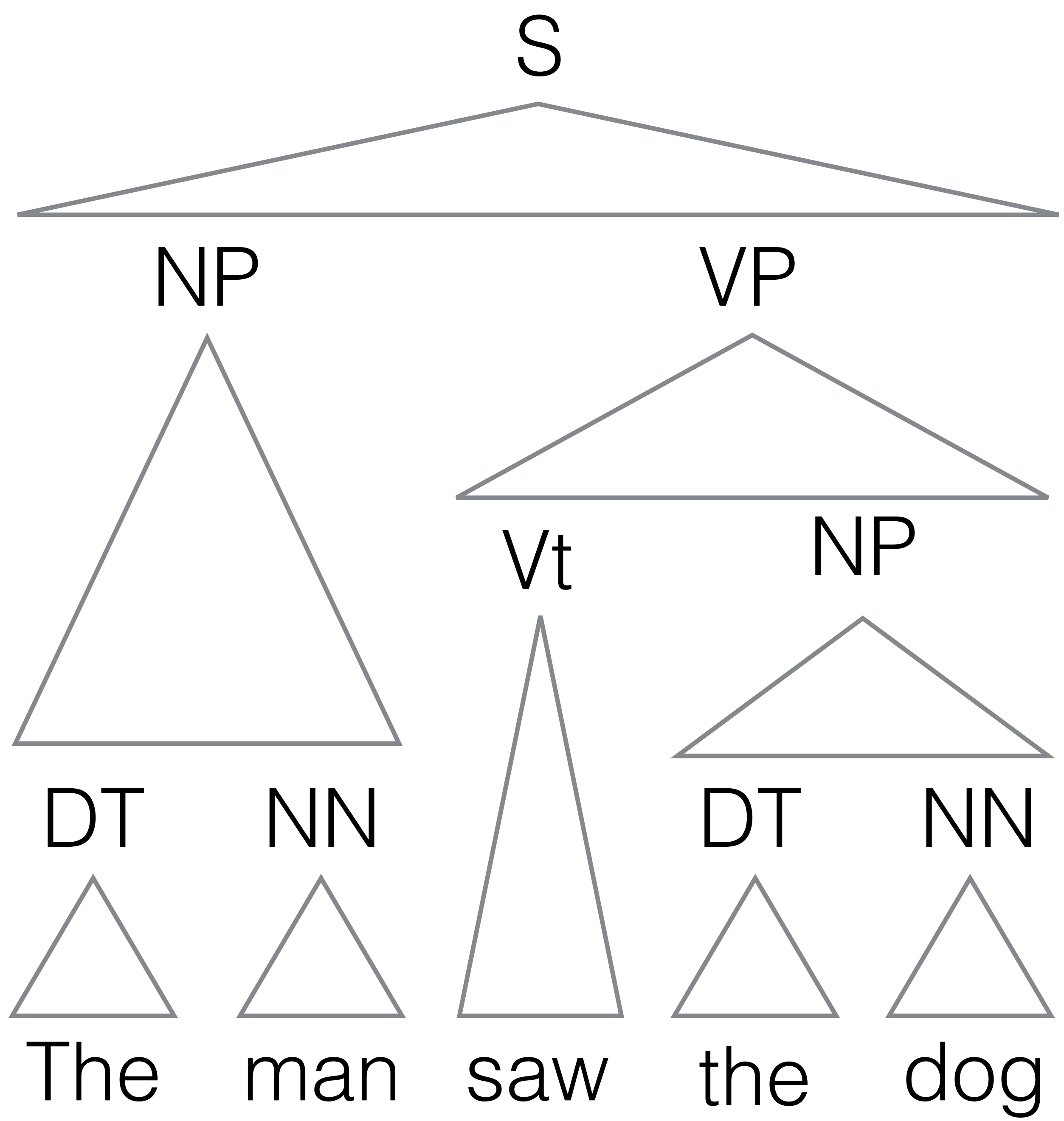

Language

A string $\omega \in \Sigma^*$ is generated/accepted by G if

$$S \Rightarrow^* \omega$$

$\Rightarrow^*$ denotes a sequence of rule applications

Language of G

$$L(G) = \{\omega: S \Rightarrow^* \omega\} \subseteq \Sigma^*$$

Chomsky Normal Form

Every CFG is weakly equivalent to another such that

- $X \rightarrow Y\ Z$ where $X, Y, Z \in \mathscr{V}$
- $X \rightarrow w$ where $w \in \Sigma$
- and possibly $S \rightarrow \varepsilon$

[Hopcroft and Ullman, 1979]

Parsing as Deduction

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Deductive process to prove claims about grammaticality

[Shieber et al., 1995]

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- dynamic program follows directly

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Deductive process to prove claims about grammaticality

[Shieber et al., 1995]

- focus on strategy rather than implementation
- soundness/completeness easier to prove
- complexity determined by inspection
- dynamic program follows directly
- generality

Deductive systems

Item: a statement / intermediate sound result

- formula or schemata expressed with variables

Inference rule: statement derived from existing items

- $$\frac{A_1 \ldots A_m}{B} \text{ (condition)}$$ where A_i and B are items
    - Ai are called antecedents
    - B is called consequent

Deductive program

Axioms: trivial items

- do not depend on previous statements

Goal: states that a proof exists

Proof:

- start from axioms
- exhaustively deduce items
  - never twice under the same premises
- accept if goal is proven

Bottom-up: Shift-Reduce

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The man saw the dog

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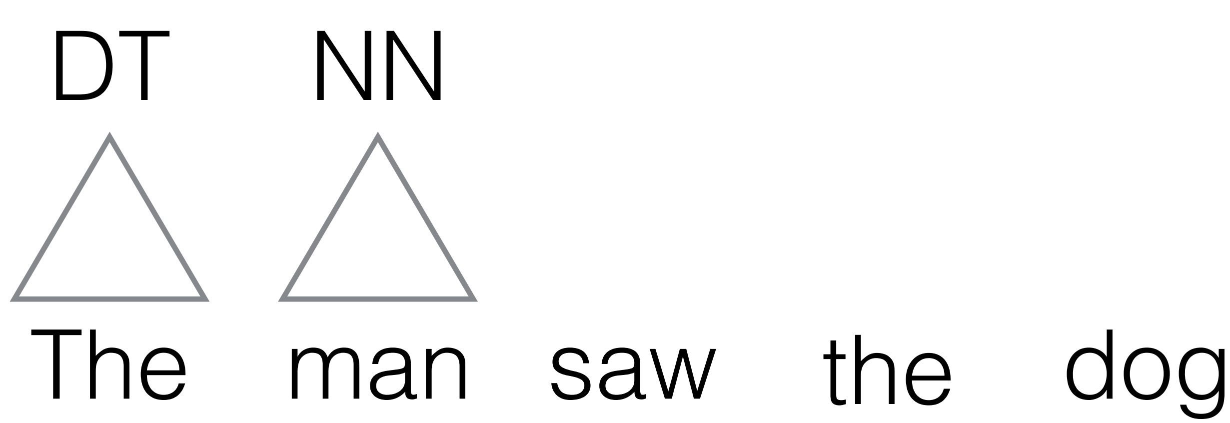

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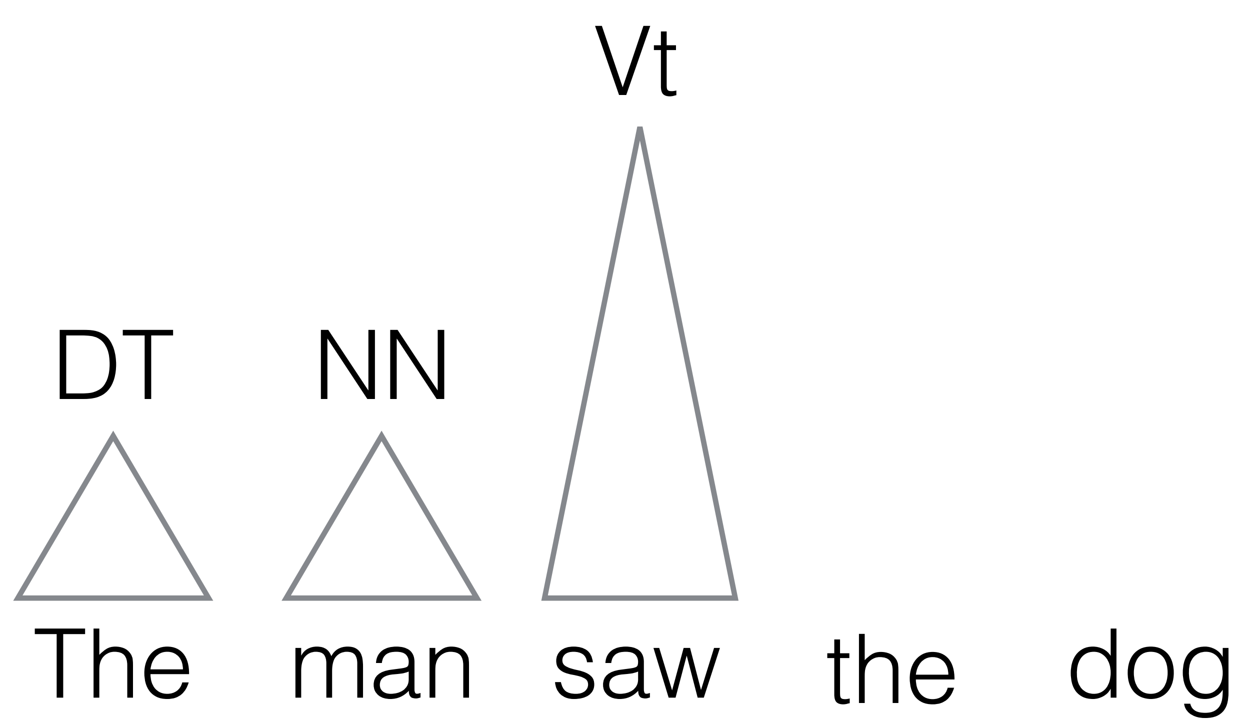

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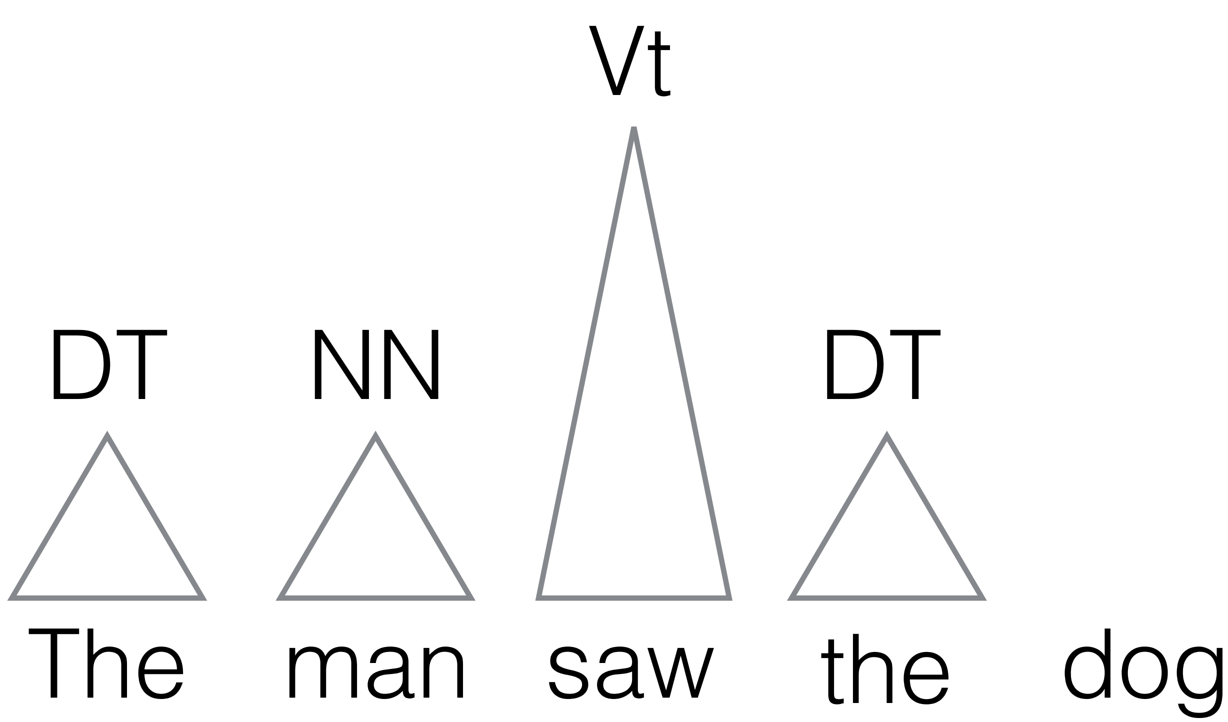

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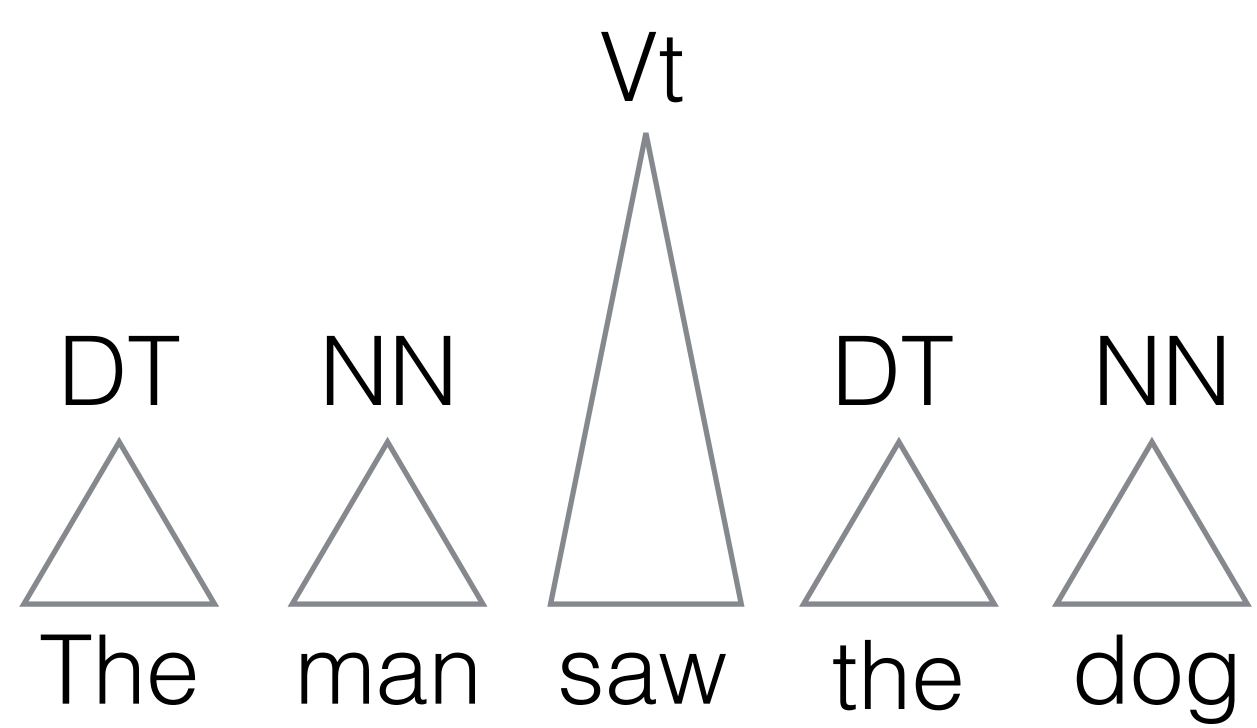

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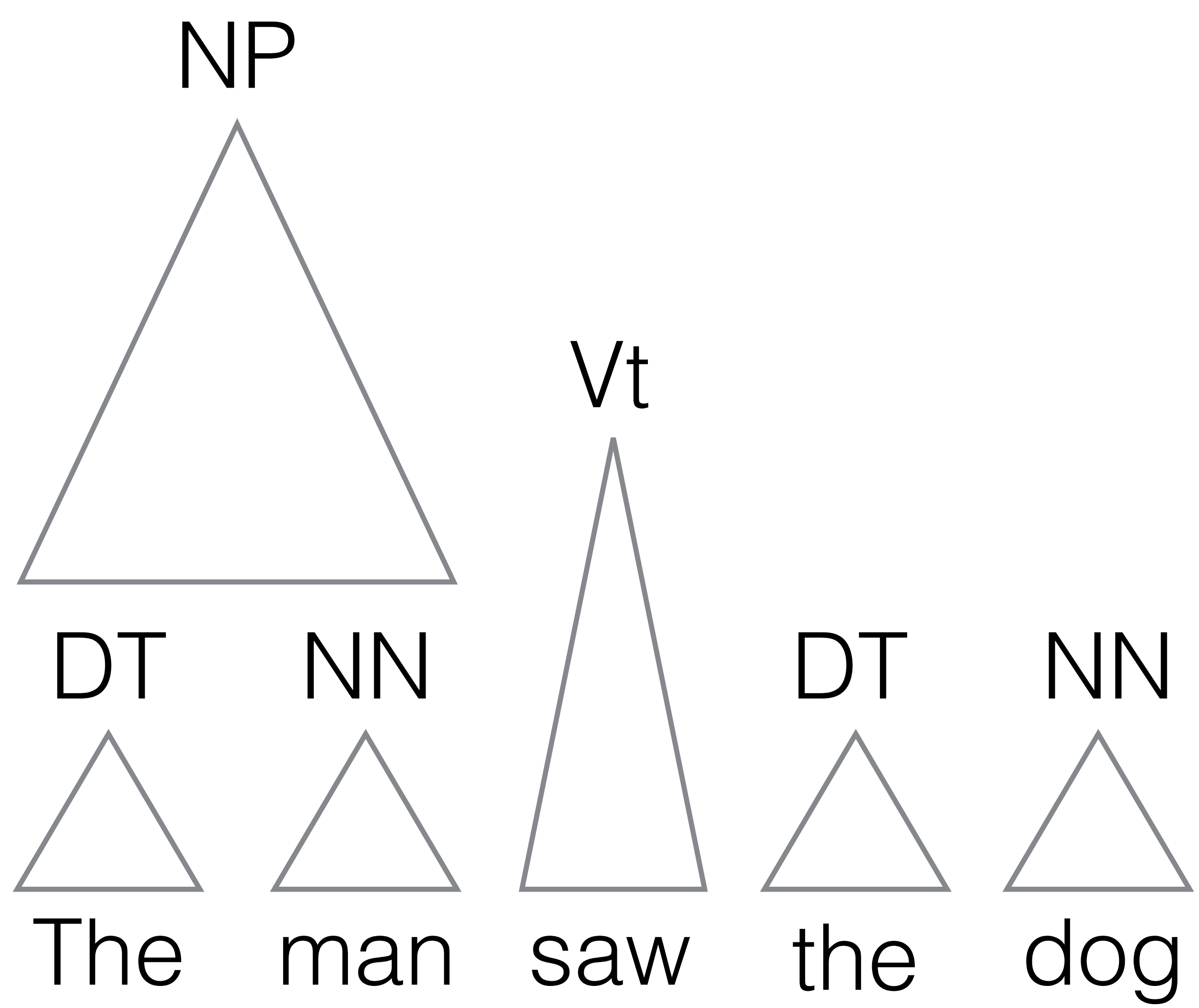

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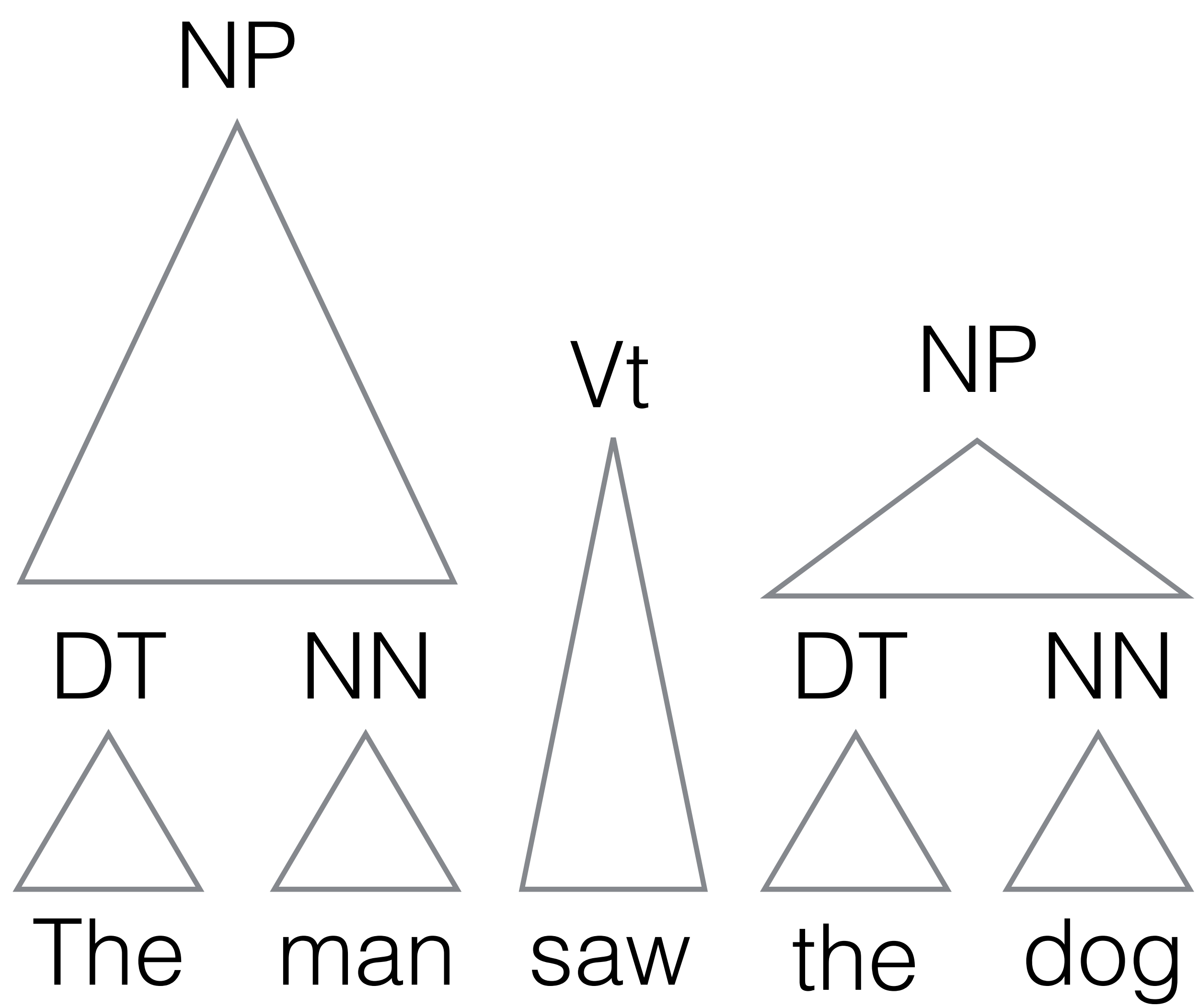

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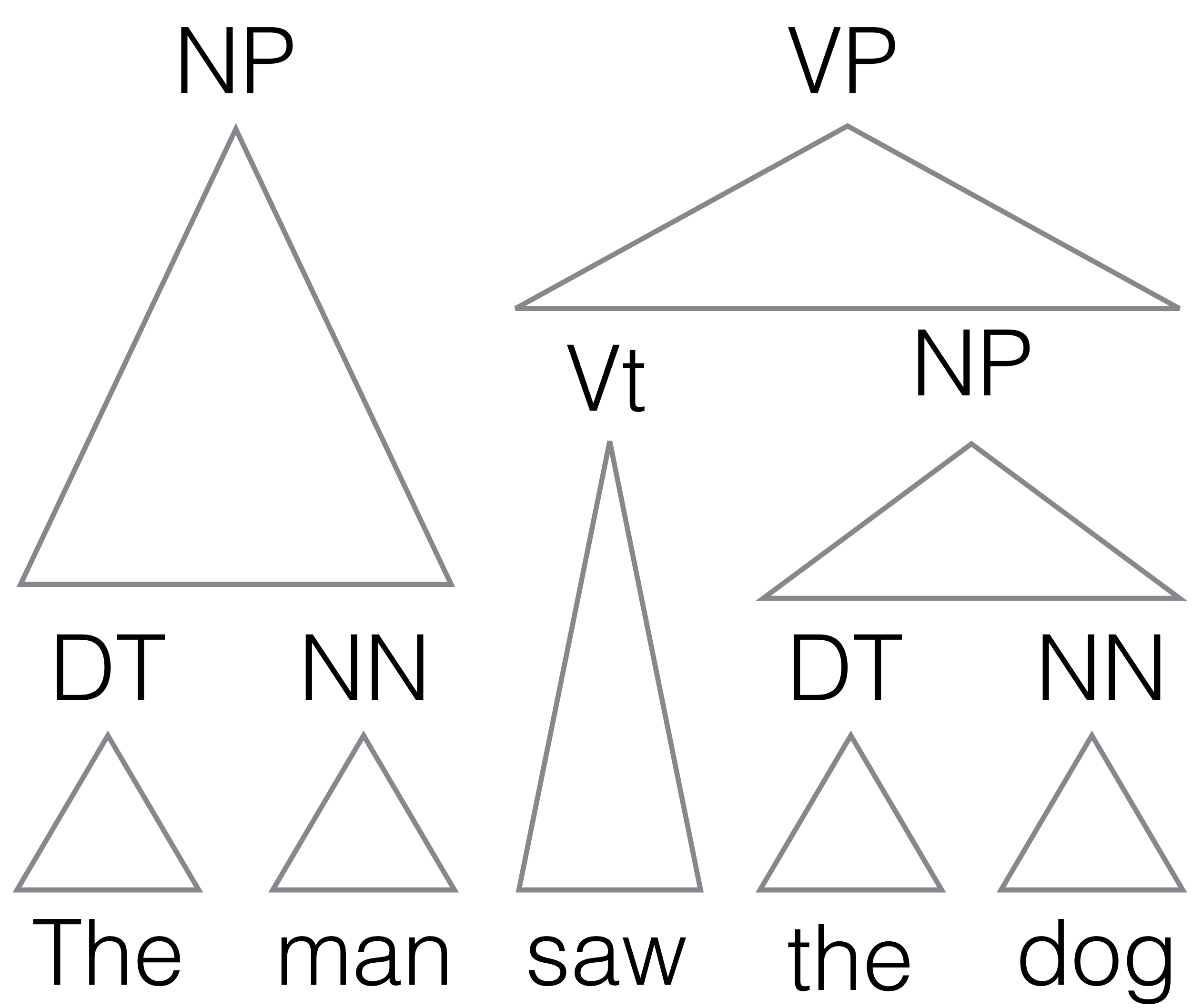

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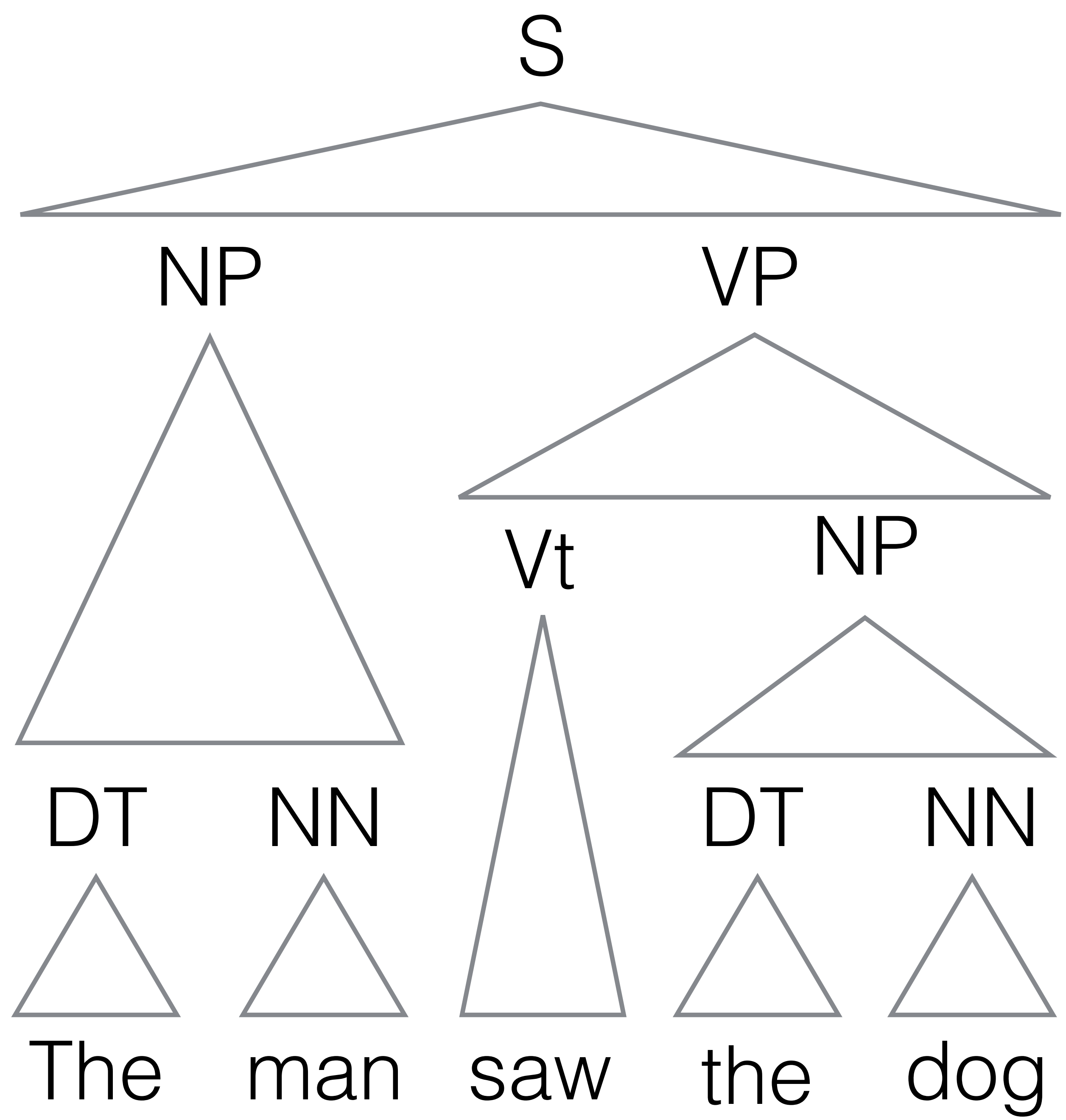

Shift-Reduce Example

Input: *the man sleeps*

S → NP VP
VP → Vi
VP → Vt NP
VP → VP PP
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
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Axiom	1	[•,0]	1
Shift: [1]	2	[the•,1]	2

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Reduce: [2]	DT → the	3	[DT•,1]	3

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Reduce: [5]	NP → DT NN	6	[NP •, 2]	6

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Reduce: [5]	NP → DT NN	6	[NP •, 2]	6
Shift: [6]		7	[NP sleeps •, 3]	7

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Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8

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Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9	[NP VP •, 3]	9

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Rule	Condition		Statement	Queue
Axiom		1	[•,0]	1
Shift: [1]		2	[the•,1]	2
Reduce: [2]	DT → the	3	[DT•,1]	3
Shift: [3]		4	[DT man •, 2]	4
Reduce: [4]	NN → man	5	[DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6	[NP •, 2]	6
Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9	[NP VP •, 3]	9
Reduce: [9]	S → NP VP	10	[S •, 3]	10

S → NP VP
VP → Vi
VP → Vt NP
VP → VP PP
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Shift-Reduce Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[•,0]	1
Shift: [1]		2	[the•,1]	2
Reduce: [2]	DT → the	3	[DT•,1]	3
Shift: [3]		4	[DT man •, 2]	4
Reduce: [4]	NN → man	5	[DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6	[NP •, 2]	6
Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9	[NP VP •, 3]	9
Reduce: [9]	S → NP VP	10	[S •, 3]	10
GOAL: [10]				∅

S → NP VP
VP → Vi
VP → Vt NP
VP → VP PP
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Shift-Reduce

Input: G and $x_1 \ldots x_n$

Item form: $[\alpha\bullet, j]$
asserts that $\alpha \Rightarrow^* x_1 \ldots x_j$ or

that $\alpha\, x_{j+1} \ldots x_n \Rightarrow^* x_1 \ldots x_n$

Axiom: $[\bullet, 0]$

Goal: $[S\bullet, n]$

$$\textsc{Shift}\ \frac{[\alpha\bullet, j]}{[\alpha\, x_{j+1}, j+1]}$$

Scan (shift)
asserts that $\alpha\, x_{j+1} \Rightarrow^* x_1 \ldots x_j\, x_{j+1}$

$$\textsc{Reduce}\ \frac{[\alpha\,\beta\bullet, j]}{[\alpha\, B, j]}\ B \rightarrow \beta \in \mathcal{R}$$

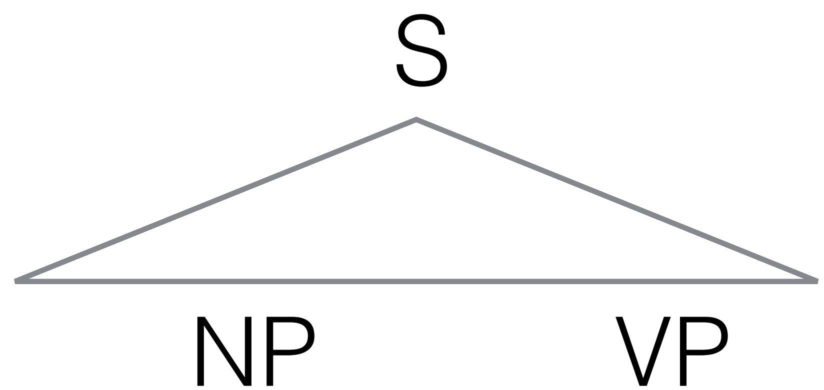
Complete (reduce)
asserts that $\alpha B \Rightarrow^* x_1 \ldots x_j$

Top-down: Predict-Scan

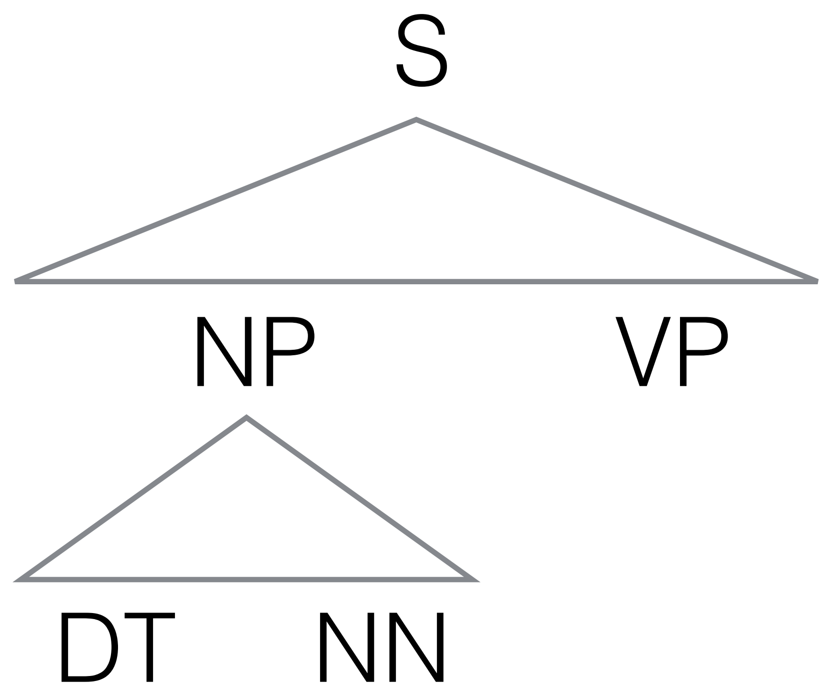
Top-down: Predict-Scan

S

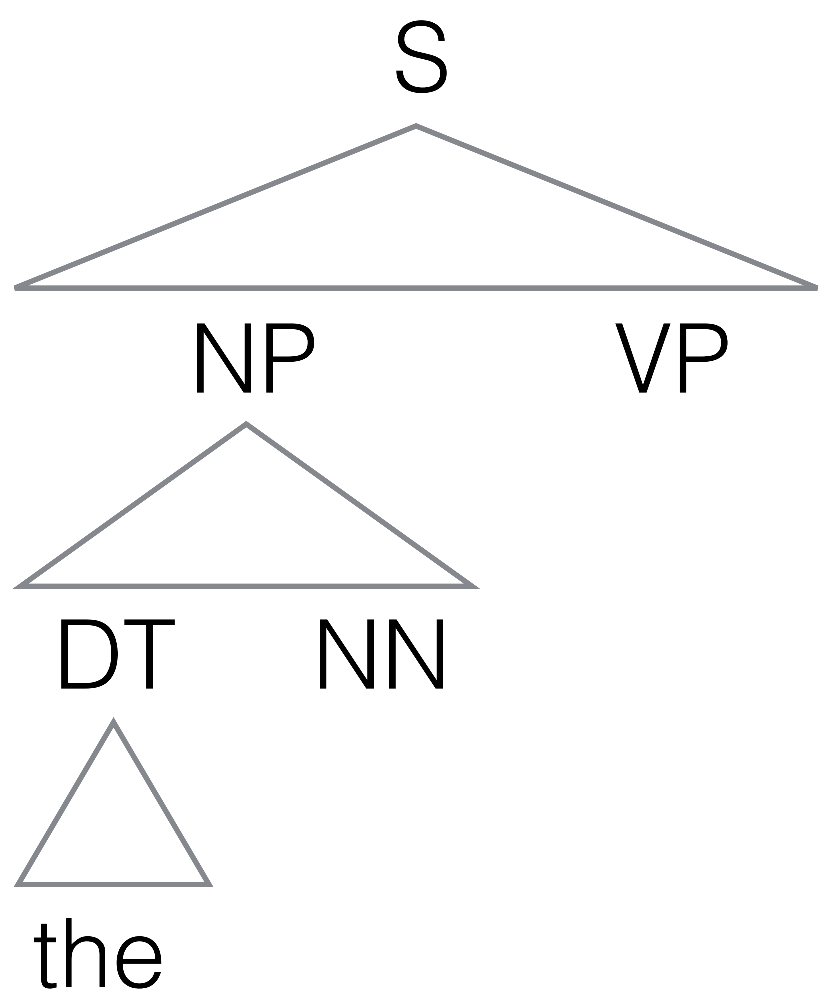
Top-down: Predict-Scan



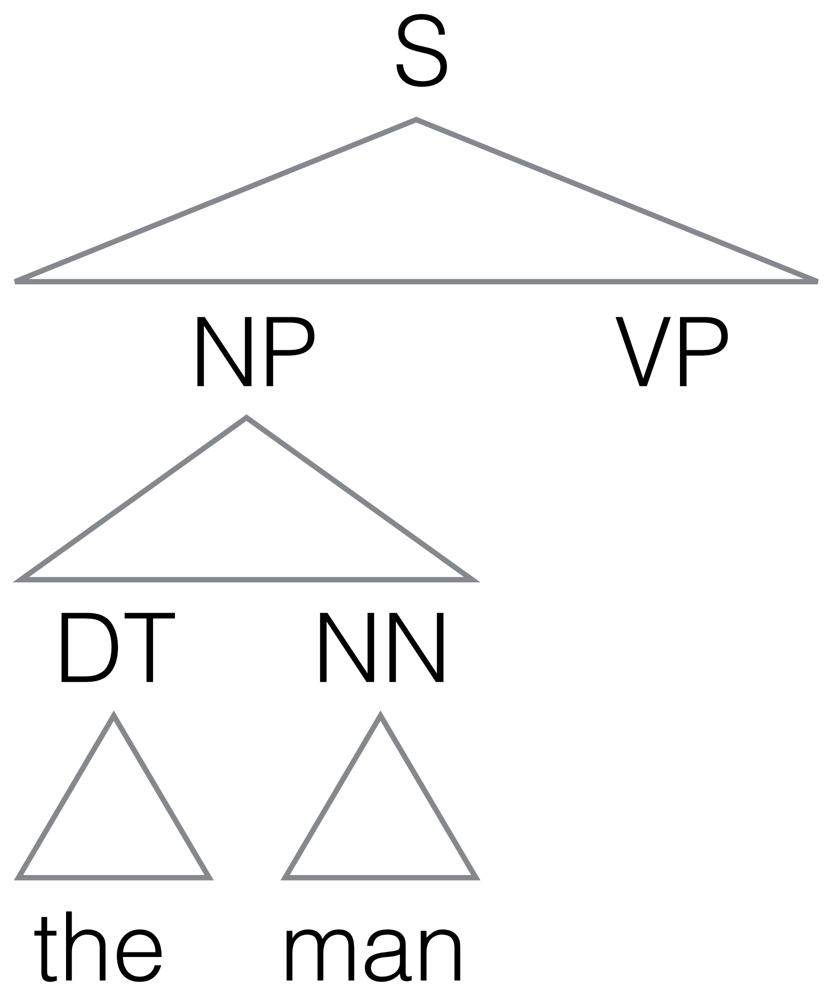
Top-down: Predict-Scan



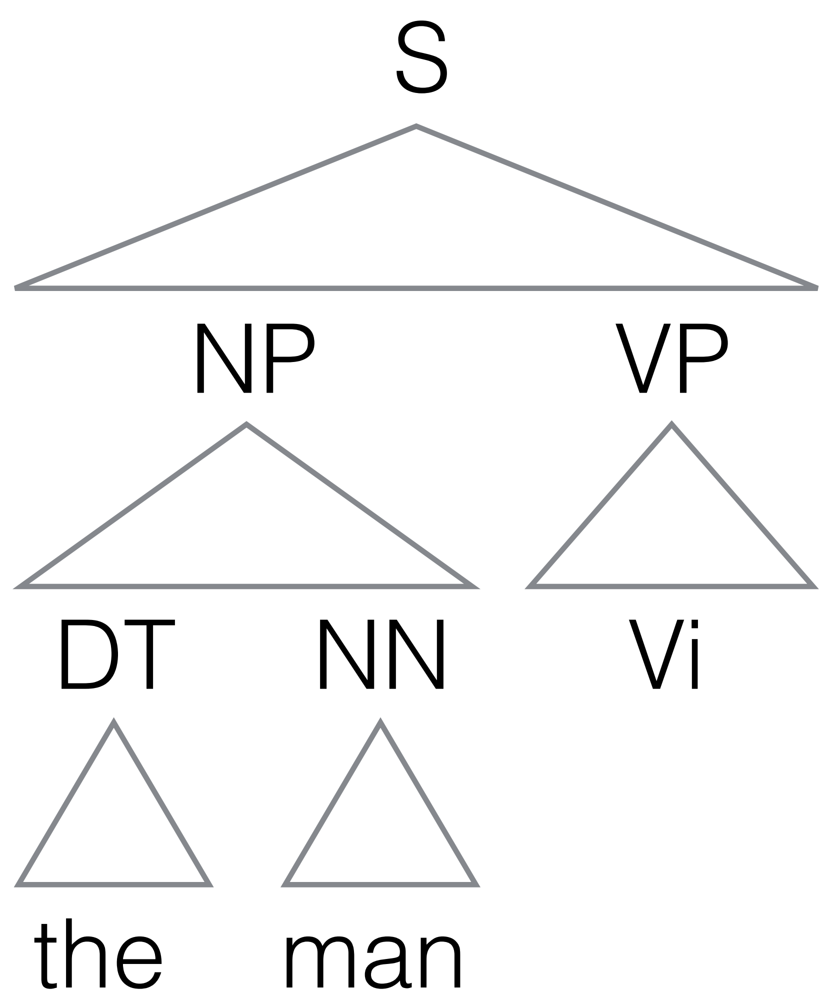
Top-down: Predict-Scan



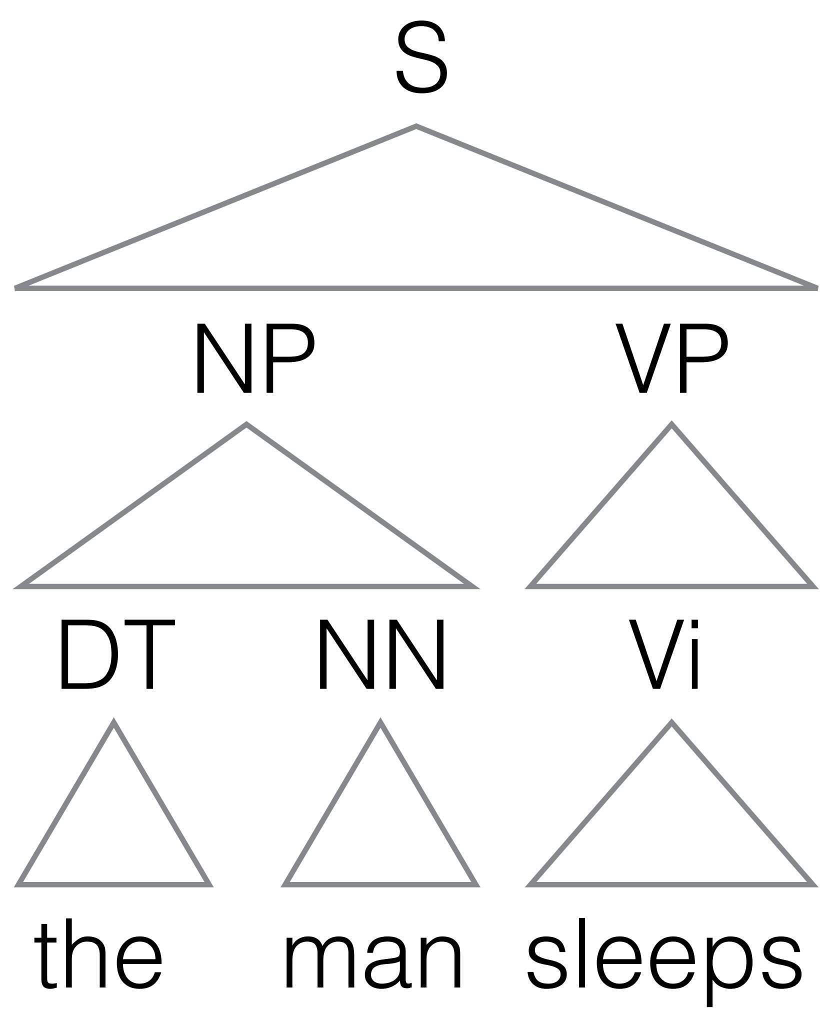
Top-down: Predict-Scan



Top-down: Predict-Scan



Top-down: Predict-Scan



Top-Down Example

Input: *the man sleeps*

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10
[9]				10

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10
[9]				10

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10
[9]				10
Scan: [10]		11	[•, 3]	11

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10
[9]				10
Scan: [10]		11	[•, 3]	11
GOAL: [11]				∅

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down recognition

Input: G and $x_1 \dots x_n$

Item form: [•β, j]
asserts that $S \Rightarrow^* x_1 \dots x_j \; \beta$

Axiom: [•S,0]

Goal: [•,n]

Scan
asserts that $S \Rightarrow^* x_1 \dots x_j \, x_{j+1} \, \beta$

$$\textsc{Scan} \; \frac{[\bullet x_{j+1}\,\beta, j]}{[\bullet \beta, j+1]}$$

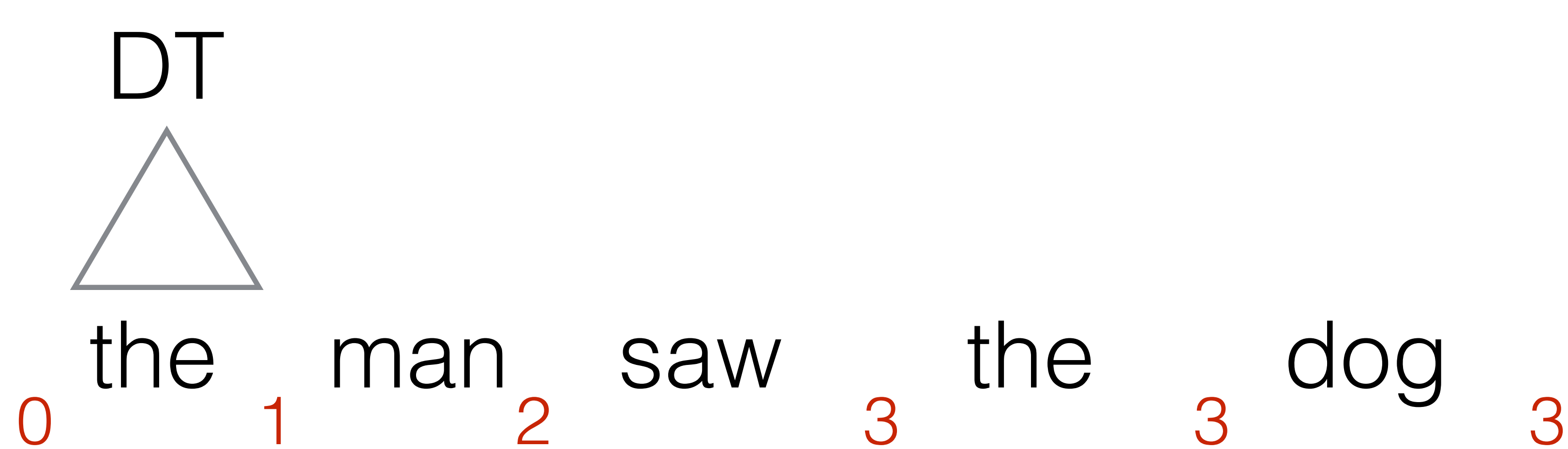
Predict
asserts that $S \Rightarrow^* x_1 \dots x_j \, B \, \beta$

$$\textsc{Predict} \; \frac{[\bullet A\,\beta, j]}{[\bullet \alpha\,\beta, j]} \; A \to \alpha \in \mathcal{R}$$

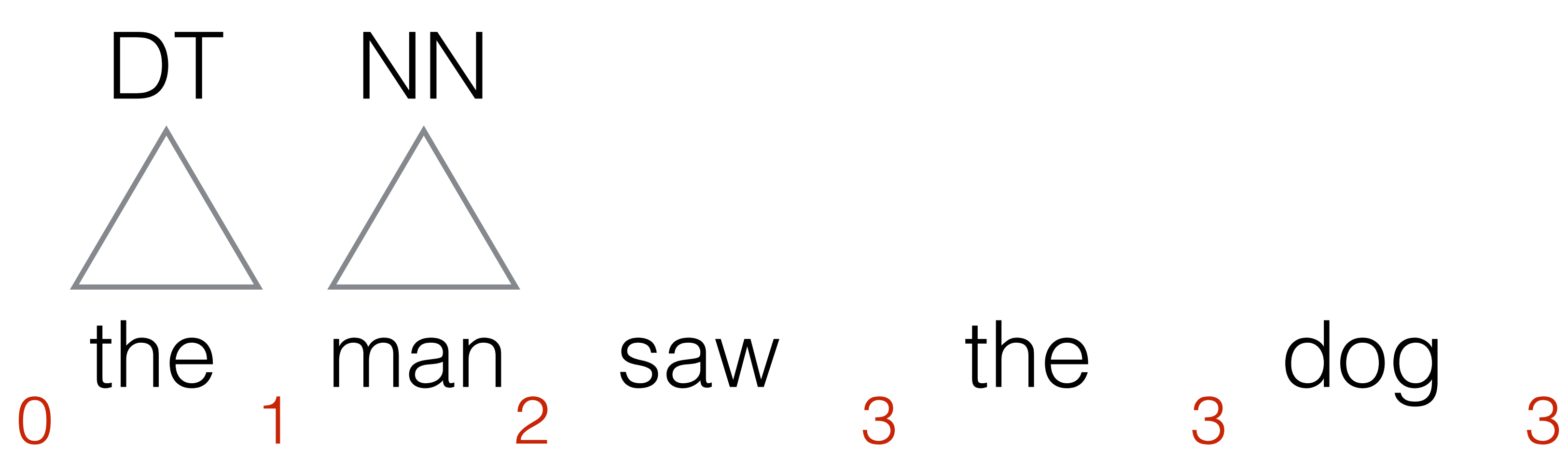
Bottom-Up for CNF: CKY

0 the 1 man 2 saw 3 the 3 dog 3

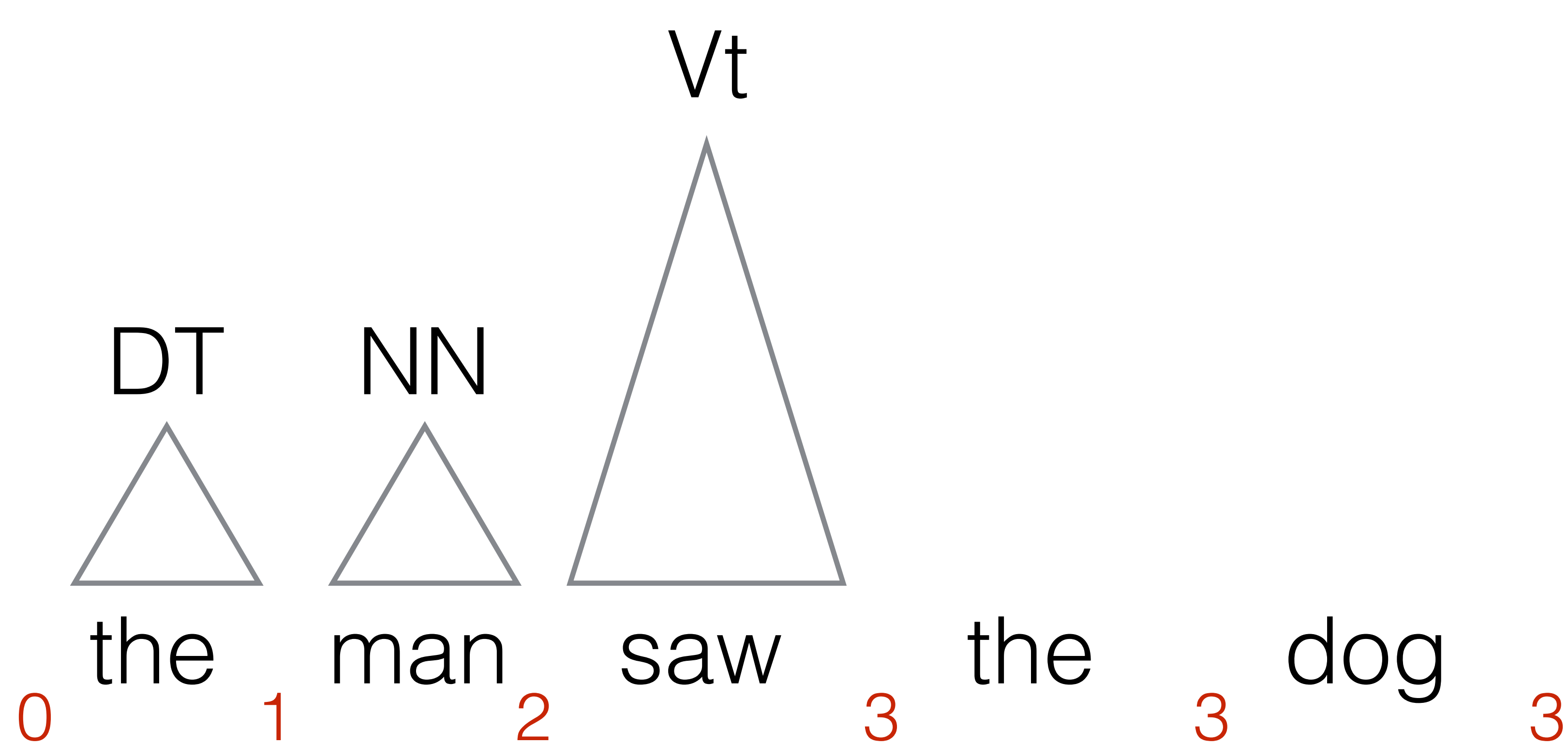
Bottom-Up for CNF: CKY



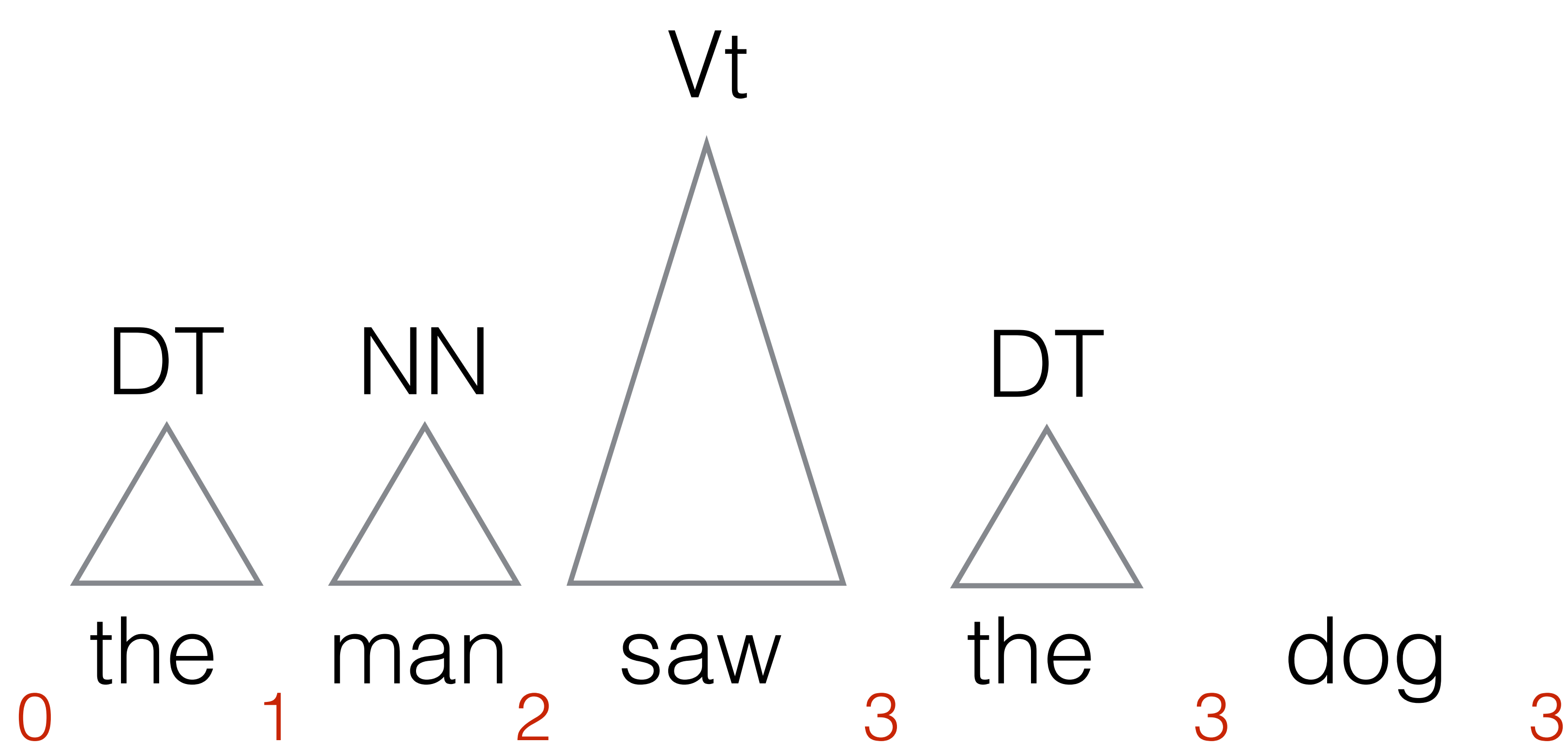
Bottom-Up for CNF: CKY



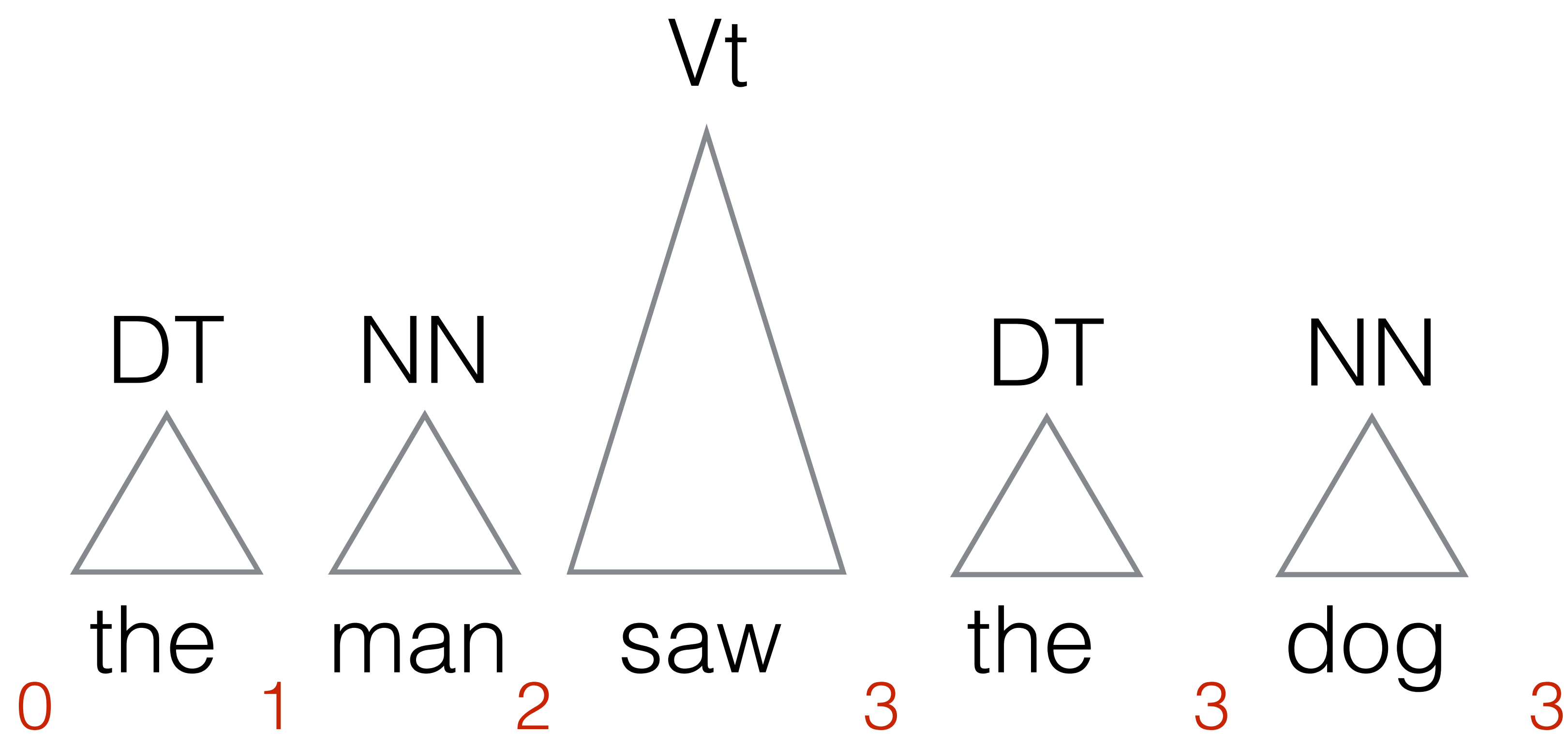
Bottom-Up for CNF: CKY



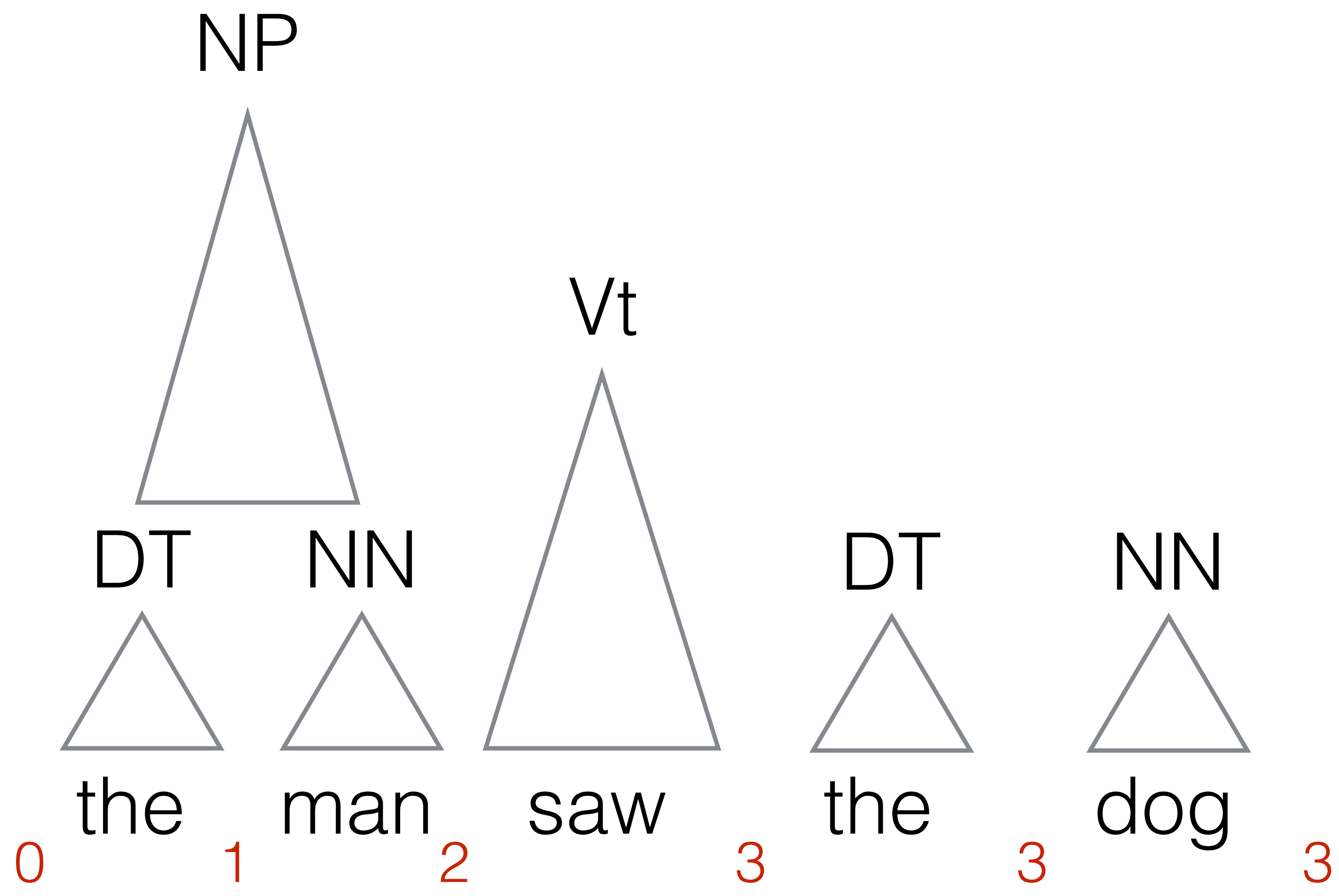
Bottom-Up for CNF: CKY



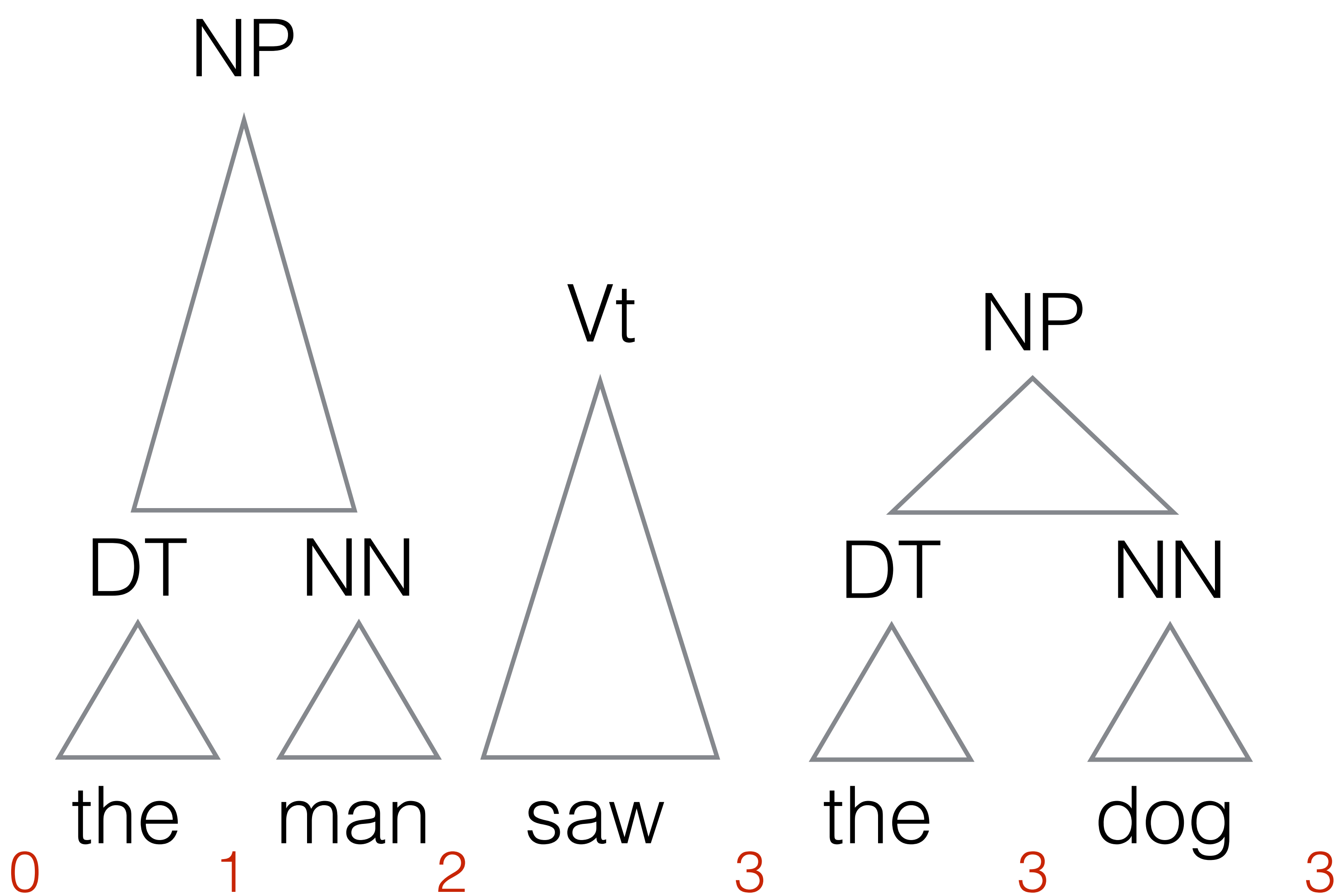
Bottom-Up for CNF: CKY



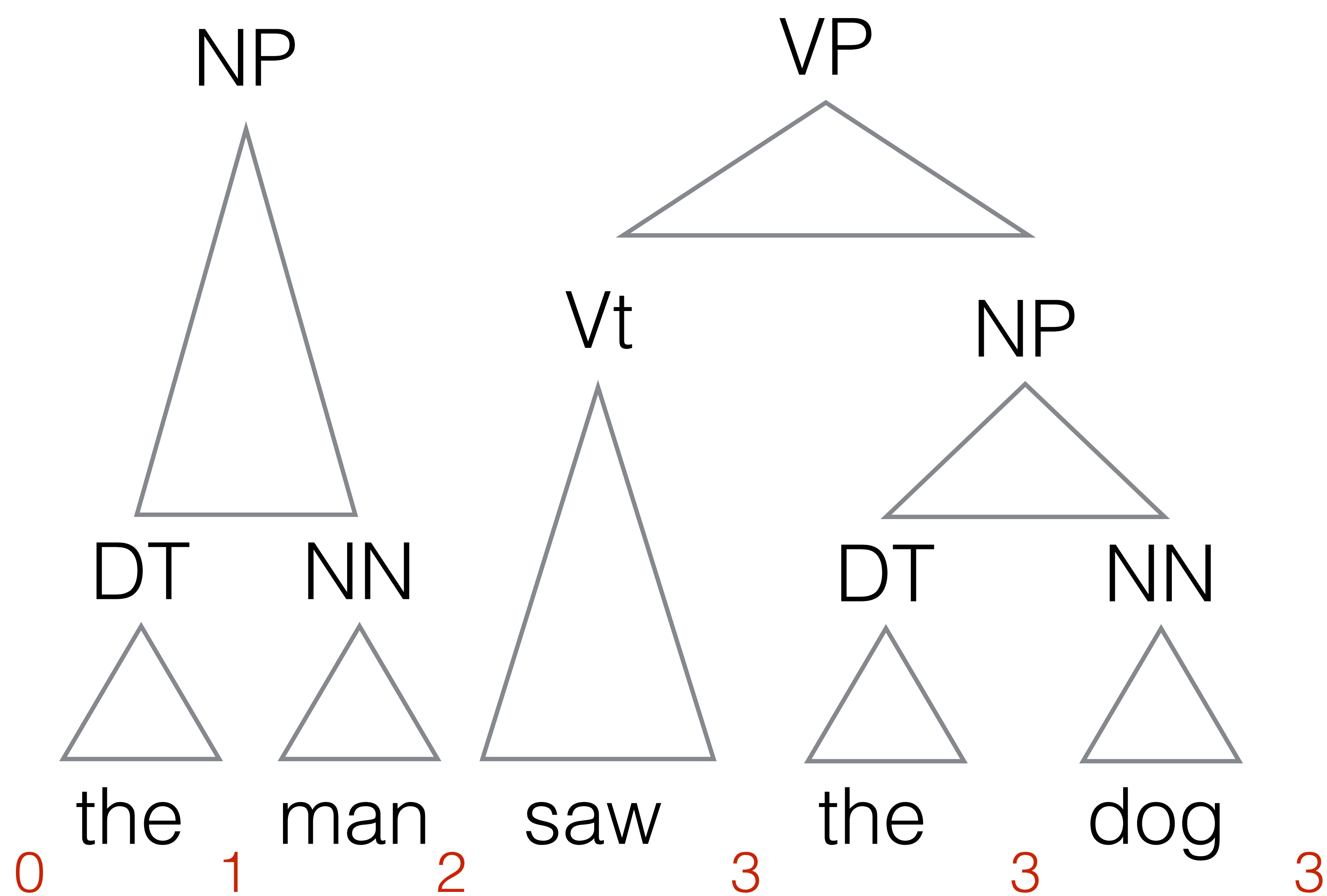
Bottom-Up for CNF: CKY



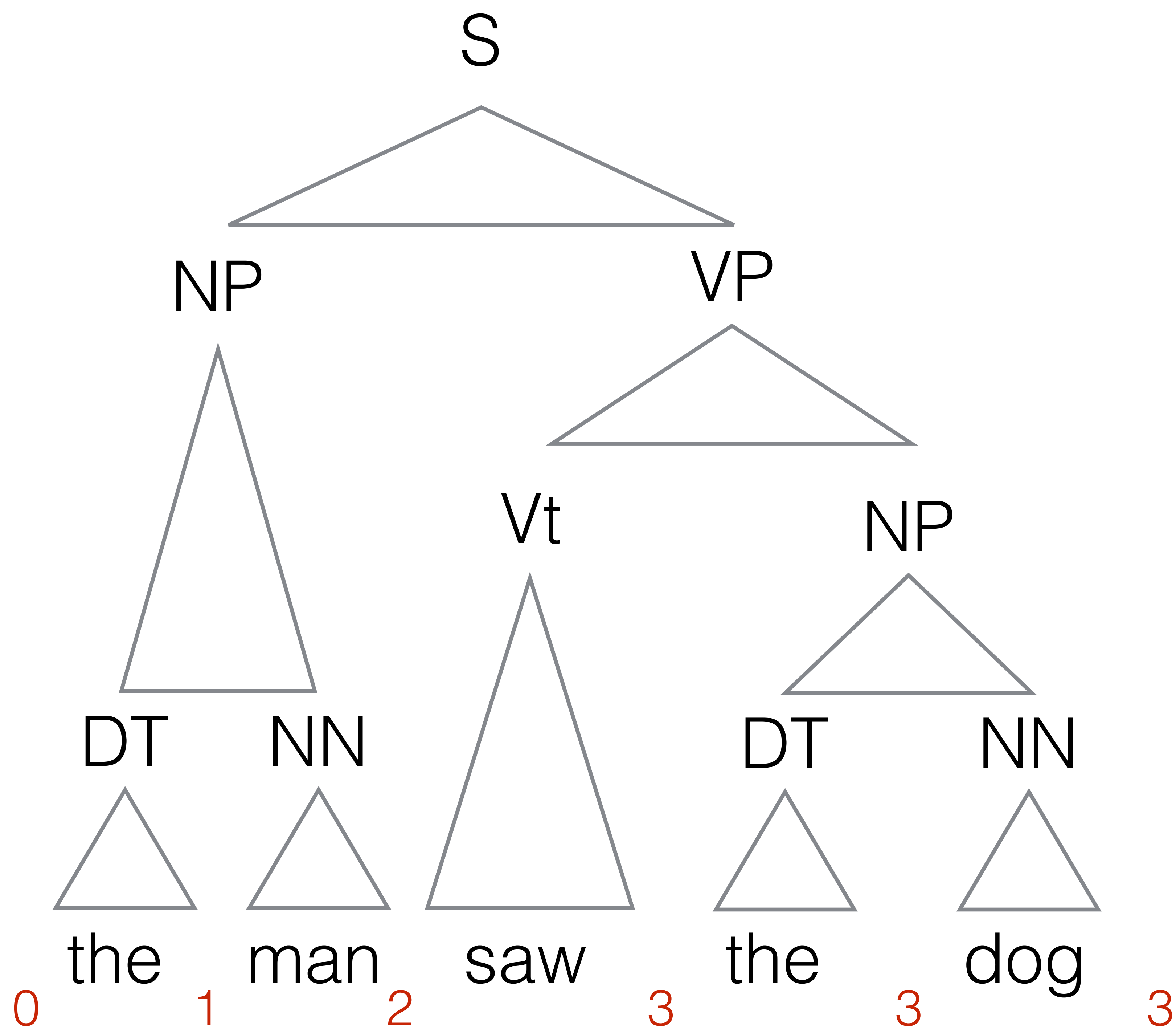
Bottom-Up for CNF: CKY



Bottom-Up for CNF: CKY



Bottom-Up for CNF: CKY



CKY - CNF only

Input: G and $s = x_1 \ldots x_n$

Item form: $[i, X, j]$
asserts that $X \Rightarrow^* x_{i+1} \ldots x_j$

Axioms: $[i, X, i+1]$ $X \rightarrow x_i \in \mathscr{R}$

Goal: $[0, S, n]$

Merge:

$$\frac{[i, A, k][k, B, j]}{[i, C, j]} \quad C \rightarrow A\,B \in \mathcal{R}$$

asserts that
$x_{i+1} \ldots x_k\ x_{k+1} \ldots x_j \Rightarrow^* x_{i+1} \ldots x_j$

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

CKY Example

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Rule	Condition	Statement	Queue	Passive

CKY Example

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NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition		Statement	Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
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VP → Vt NP	NN → man
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NP → NP PP	DT → the
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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	

CKY Example

Input: *the man saw the dog*

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VP → Vt NP	NN → man
VP → VP PP	NN → dog
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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
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VP → Vt NP	NN → man
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NP → DT NN	NN → telescope
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Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2

CKY Example

Input: *the man saw the dog*

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~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3

CKY Example

Input: *the man saw the dog*

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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4

CKY Example

Input: *the man saw the dog*

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~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
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Rule	Condition		Statement	Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5
				7	6

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5
				7	6
Merge: [3] [7]	VP → Vt NP	8	[2, VP, 5]	8	7

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5
				7	6
Merge: [3] [7]	VP → Vt NP	8	[2, VP, 5]	8	7
Merge: [6] [8]	S → NP VP	9	[0, S, 5]	9	8

CKY Example

Input: *the man saw the dog*

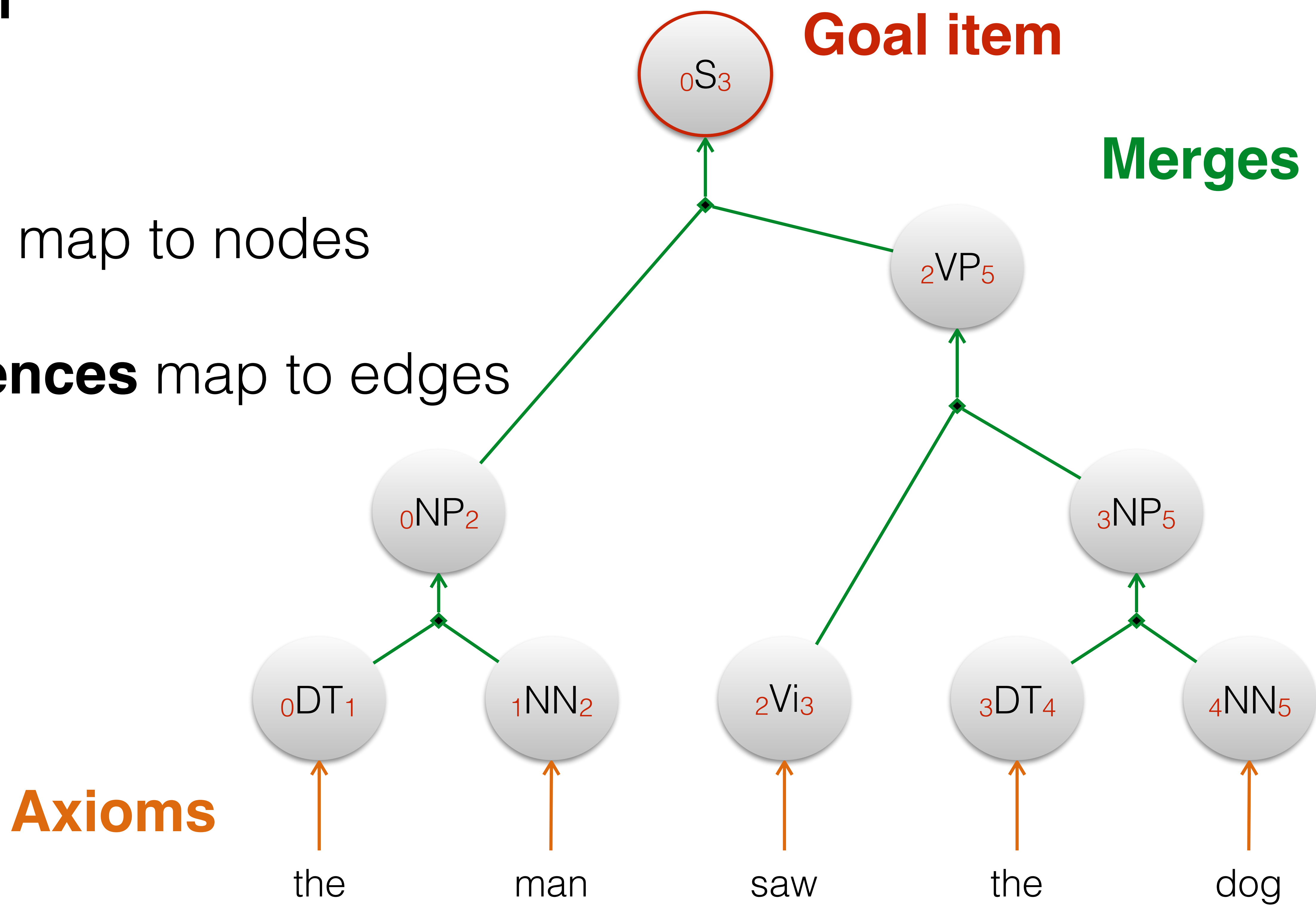
S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5
				7	6
Merge: [3] [7]	VP → Vt NP	8	[2, VP, 5]	8	7
Merge: [6] [8]	S → NP VP	9	[0, S, 5]	9	8
GOAL: [9]				∅	9

Graphical view

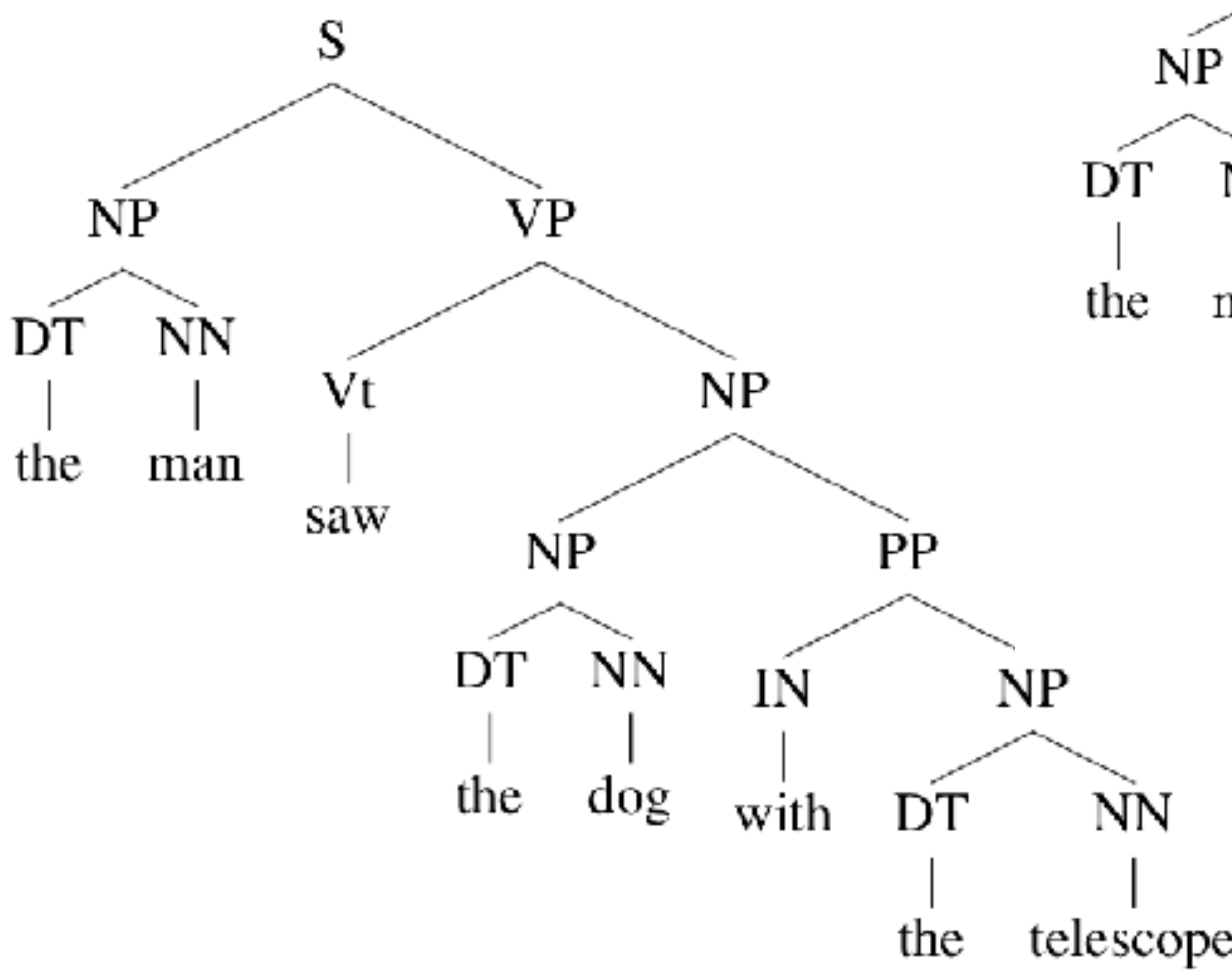
Items map to nodes

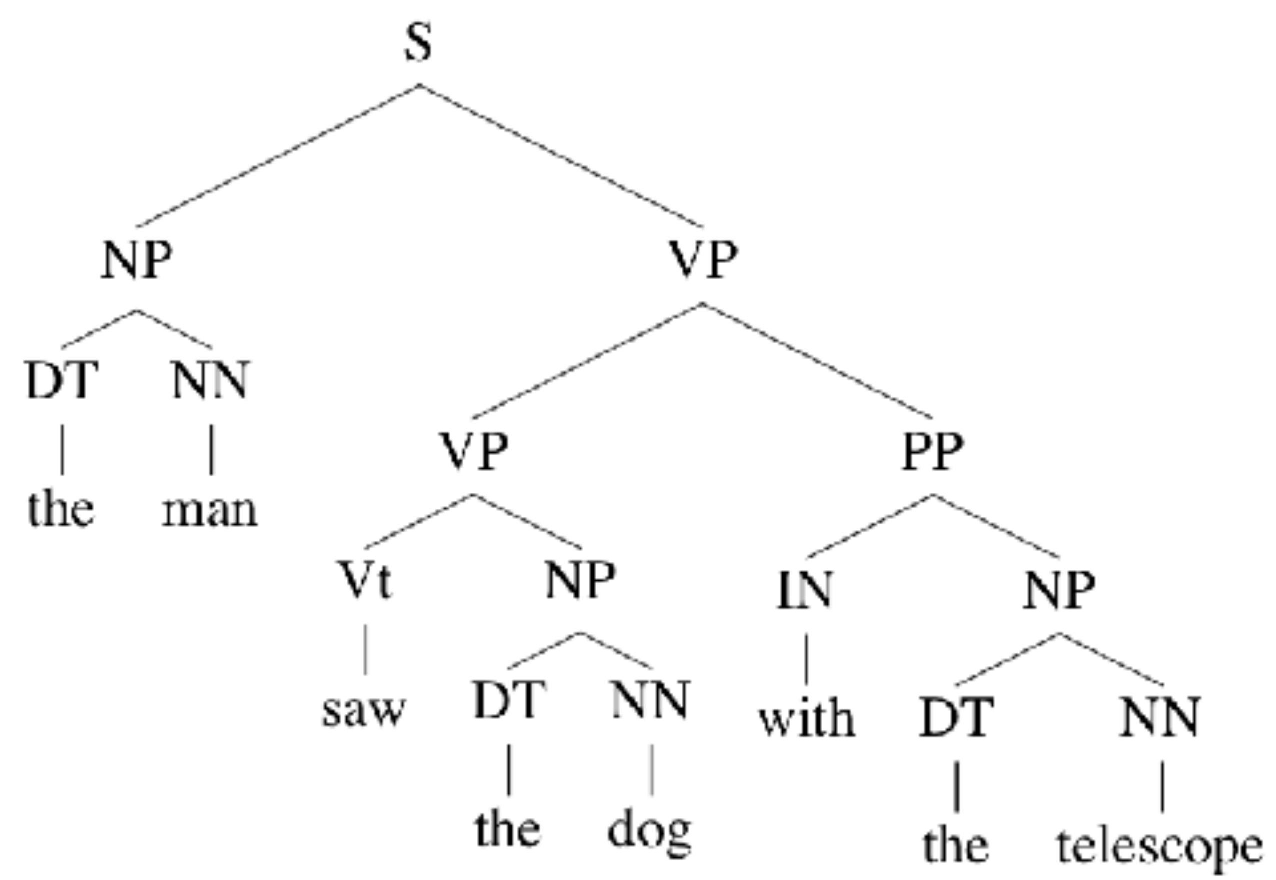
Inferences map to edges



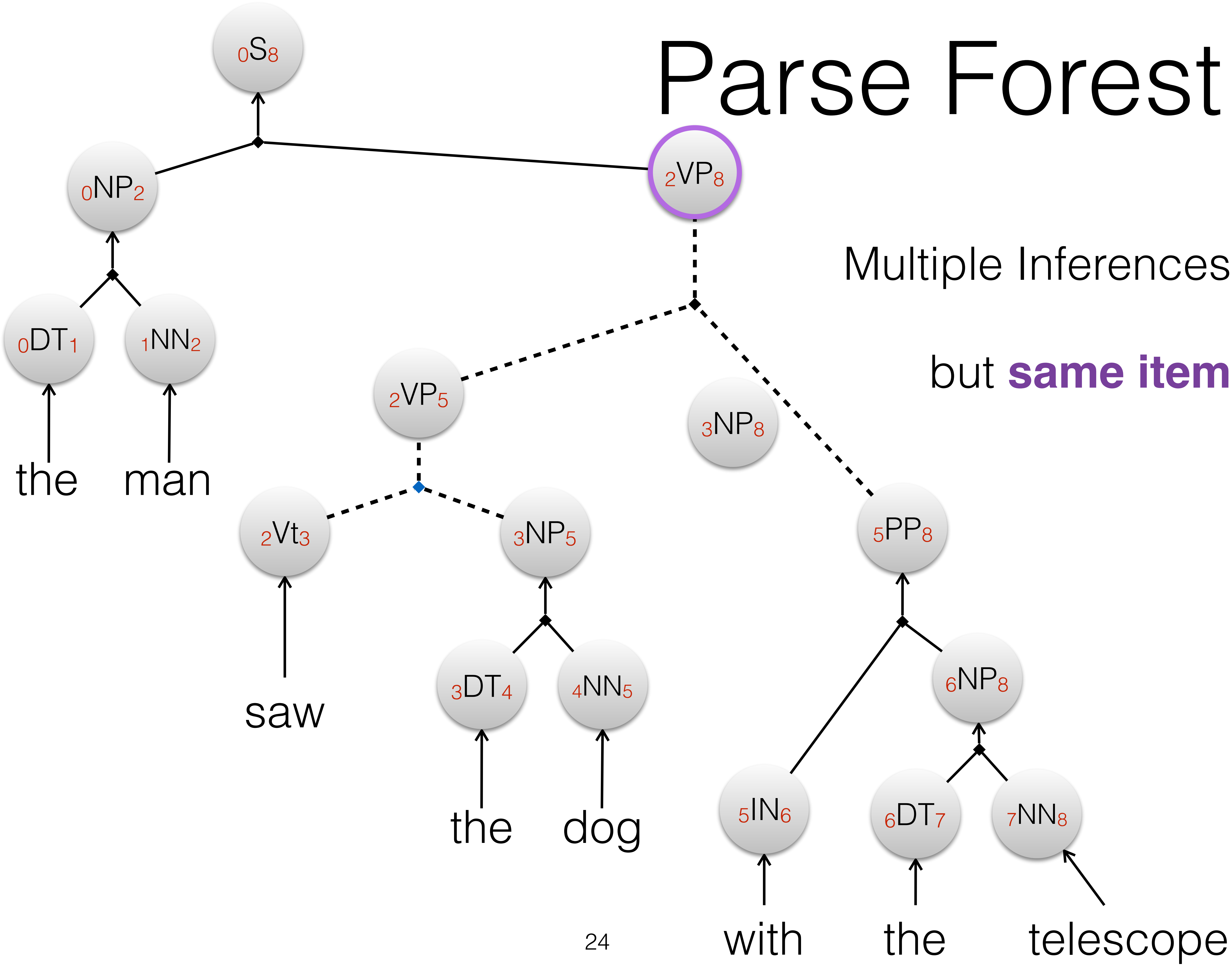
Ambiguity

Some strings may have more than one derivation in G





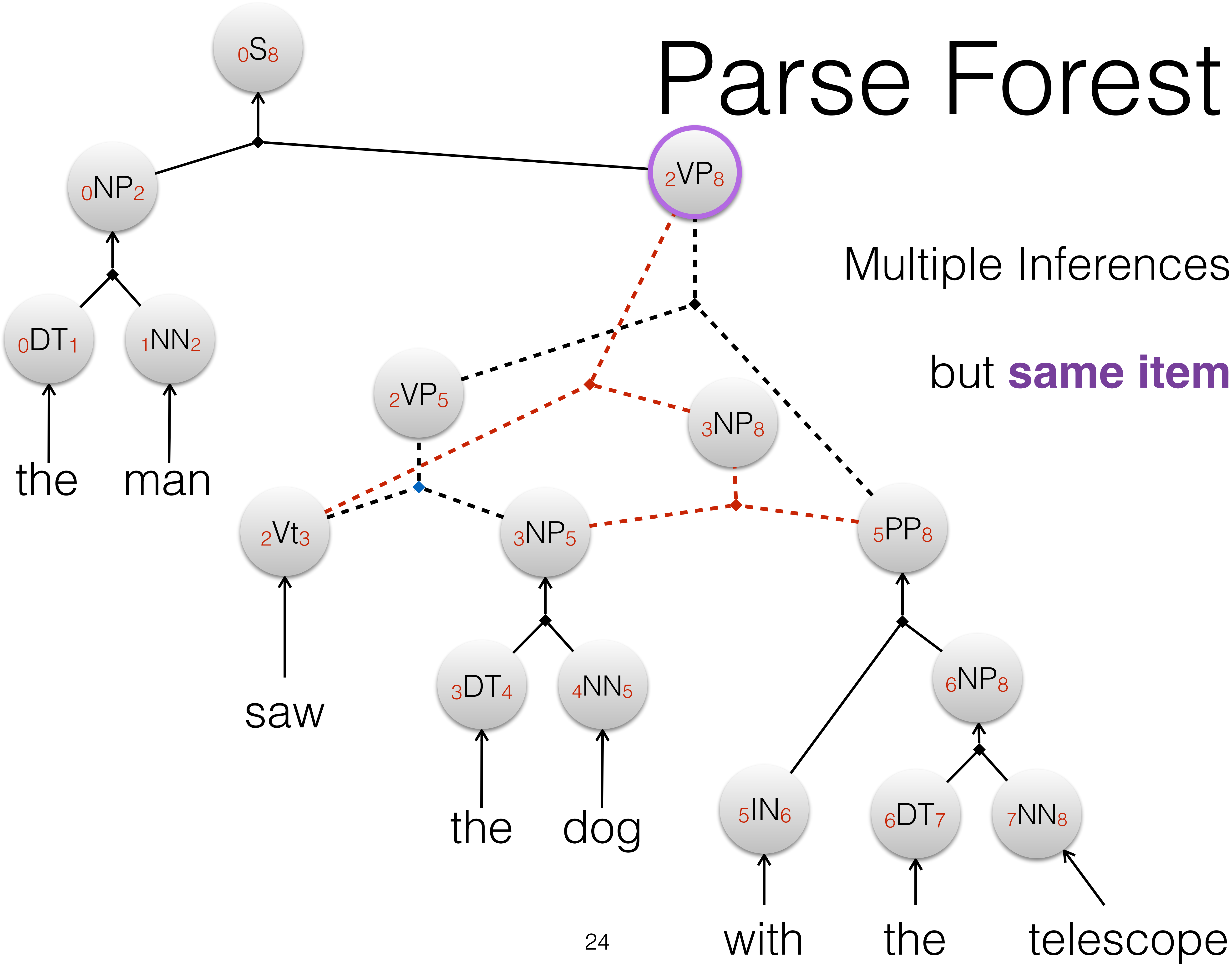
Parse Forest



Multiple Inferences

but **same item**

Parse Forest



Multiple Inferences

but **same item**

Parse Forest

Efficient representation of the whole space $T_G(\omega)$

- each and every possible tree yielding ω

Items (other than the goal) represent partial derivations

- including alternative ones

Dealing with Ambiguity

Statistical model: PCFG

- weight steps in a derivation
- induces a partial ordering over derivations
- can be used to make a decision
    - e.g. best tree under the model

Probabilistic CFG

CFG extended with parameters $0 \leq \theta_r \leq 1$

- where $r \in \mathscr{R}$ and

$$\sum_{\beta : v \rightarrow \beta \in \mathcal{R}} \theta_{v \rightarrow \beta} = 1$$

Probabilistic CFG

Distribution over trees and their yields

$$P_{DS|NM}(R_1^m = r_1^m, X_1^n = \text{yield}(r_1^m)|n, m)$$
$$= \prod_{i=1}^{m} \theta_{r_i} = \prod_{i=1}^{m} \theta_{v_i \to \beta_i}$$
$$\text{where } r_i \text{ corresponds to } v_i \to \beta_i$$

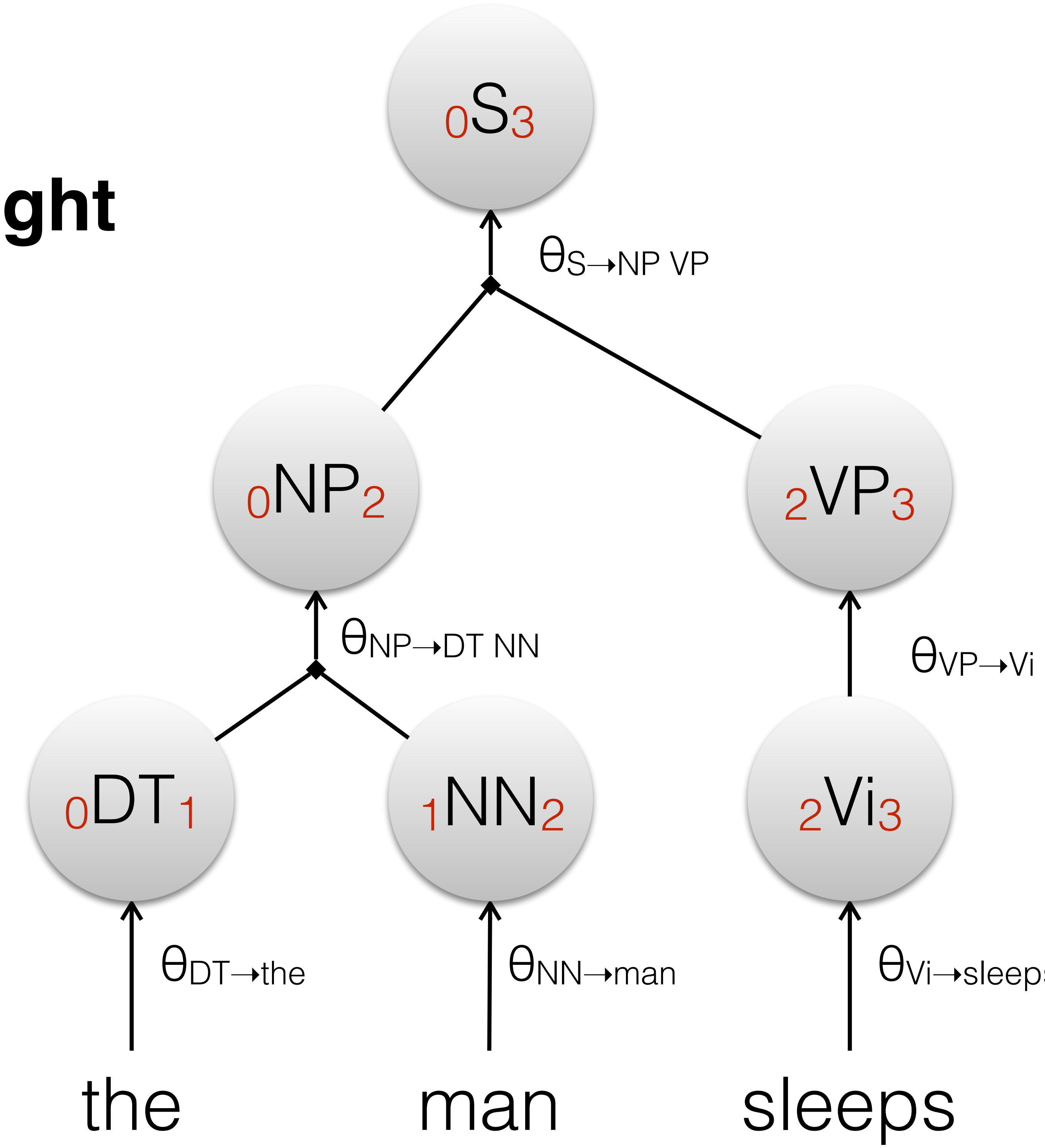
Joint Distribution

Each inference gets a **weight**

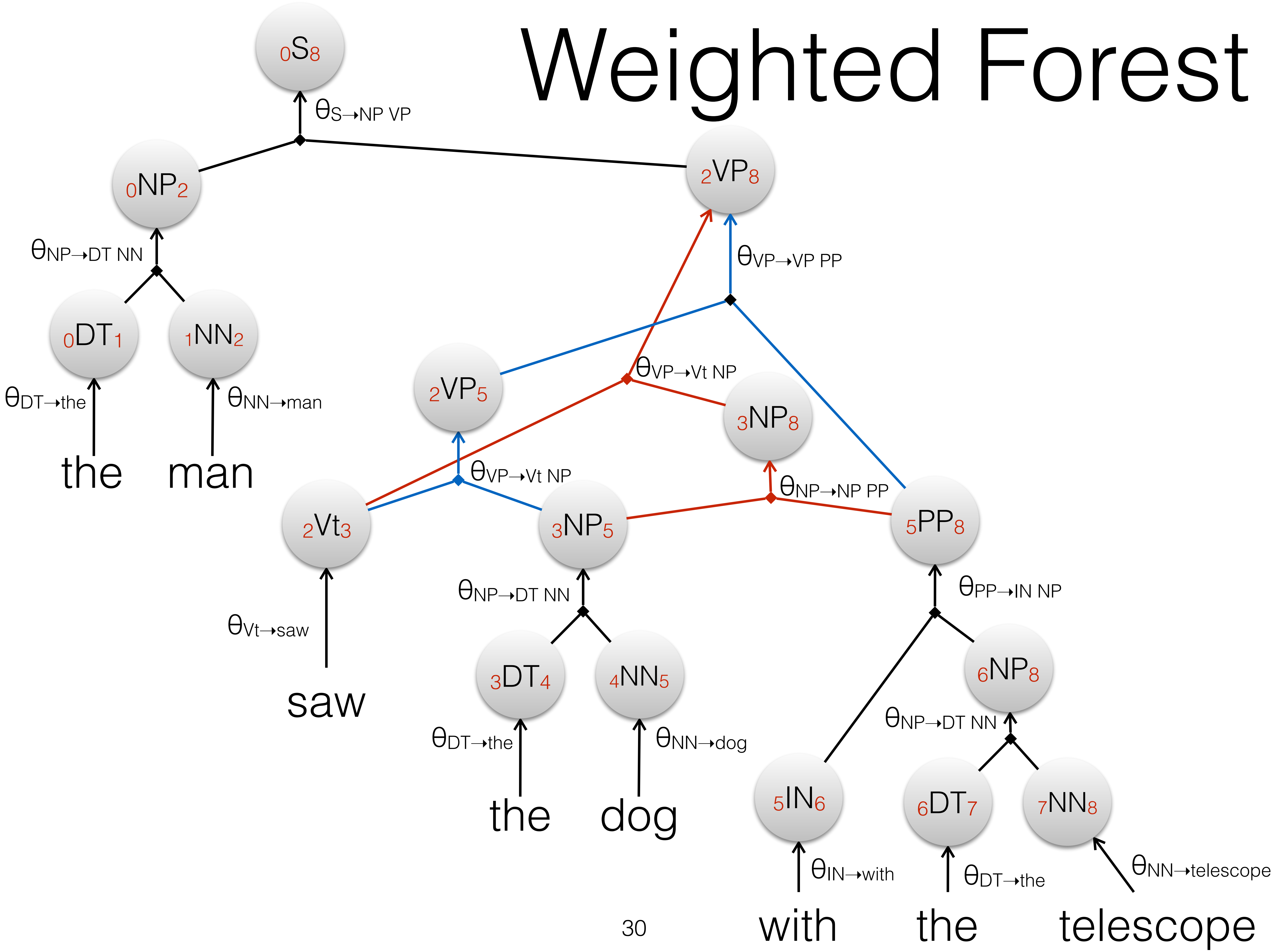
i.e. categorical parameter

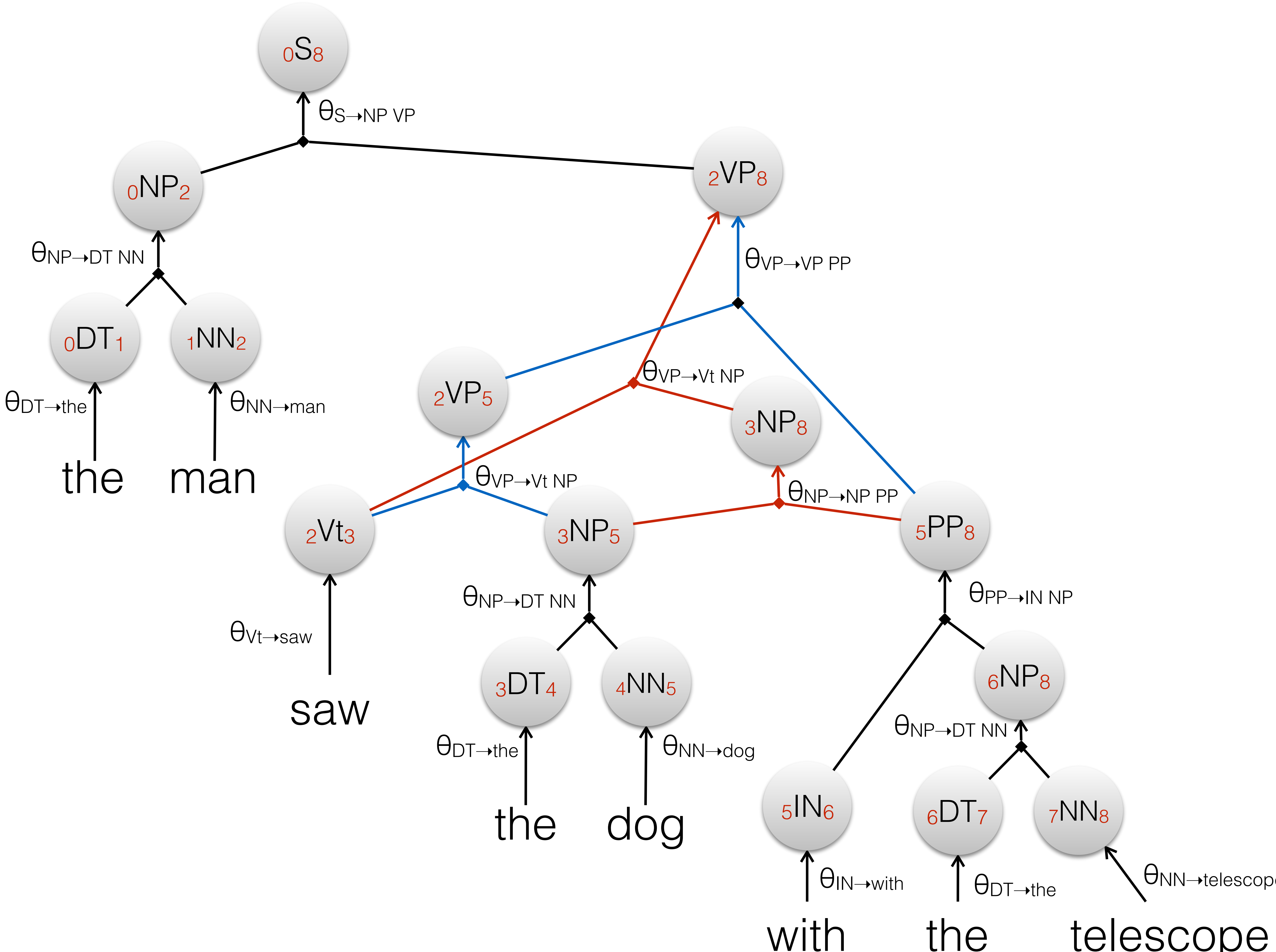
$$\theta_{X \to \beta}$$

of the underlying rule



Weighted Forest





0S8
θS→NP VP
0NP2
2VP8
θNP→DT NN
θVP→VP PP
0DT1
1NN2
θDT→the
θNN→man
the
man
2VP5
θVP→Vt NP
3NP8
θVP→Vt NP
θNP→NP PP
2Vt3
3NP5
5PP8
θNP→DT NN
θPP→IN NP
θVt→saw
saw
3DT4
4NN5
6NP8
θDT→the
θNN→dog
θNP→DT NN
the
dog
5IN6
6DT7
7NN8
θIN→with
θDT→the
θNN→telescope
with
the
telescope

Marginal Probability

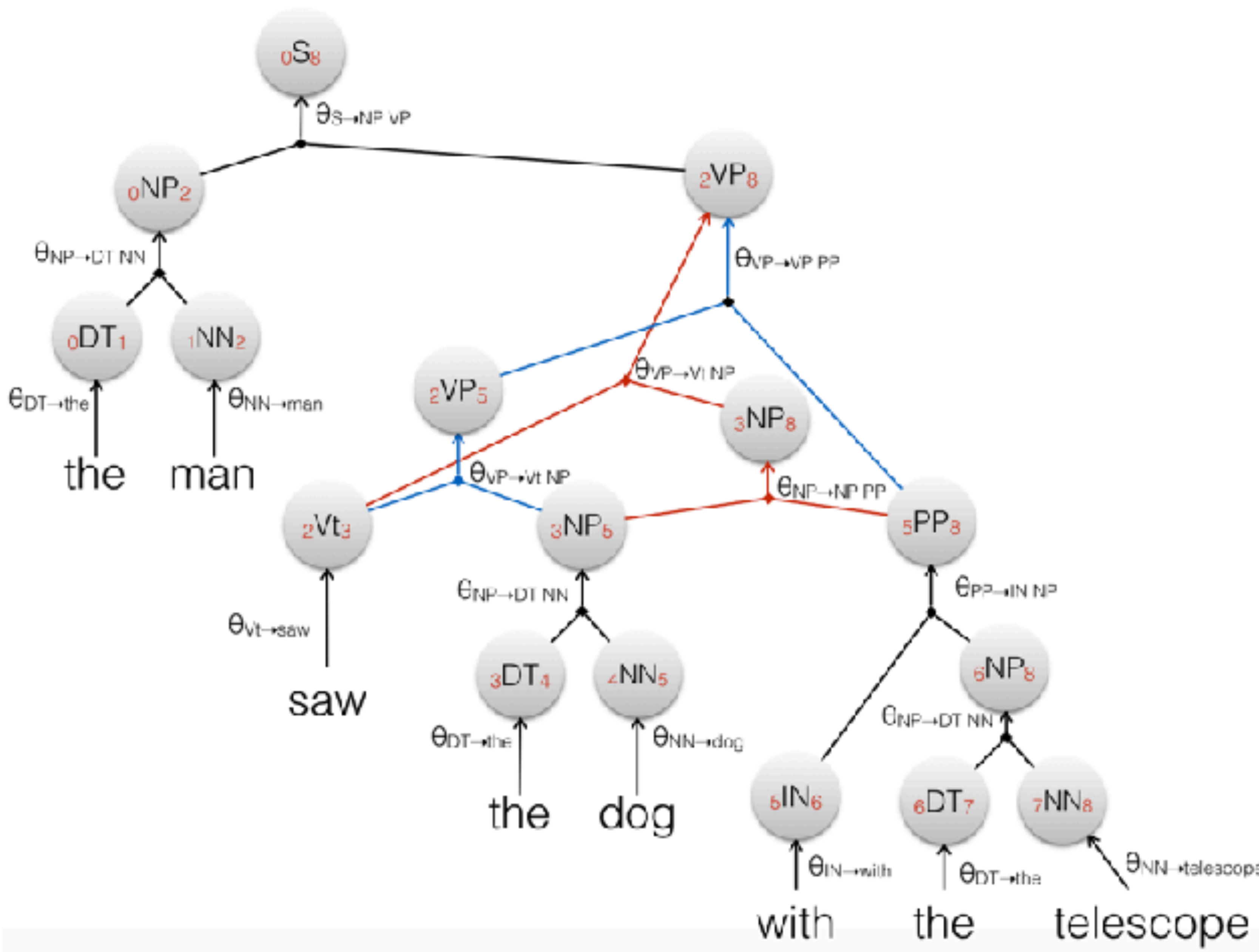
$$P_{S|n}(x_1^n|n) = I({}_0\mathrm{S}_n) = \sum_{r_1^m \in \mathcal{G}(x_1^n)} \prod_{i=1}^{m} \theta_{v_i \to \beta_i}$$

Marginal Probability

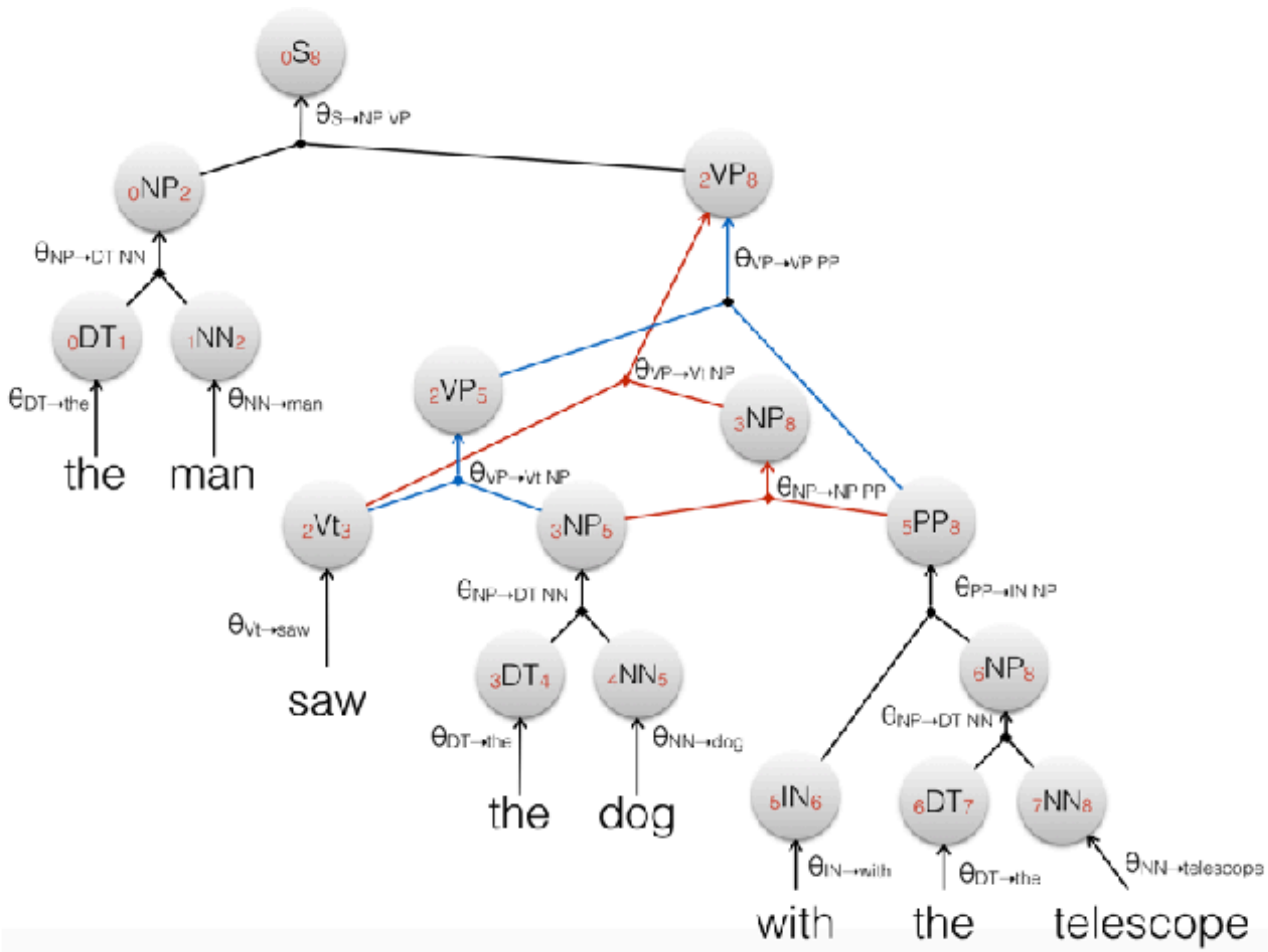
Let the goal item **stand** for the sentence. What's its (marginal/inside) probability $I({}_0S_8)$?

$$P_{S|n}(x_1^n|n) = I({}_0\mathrm{S}_n) = \sum_{r_1^m \in \mathcal{G}(x_1^n)} \prod_{i=1}^{m} \theta_{v_i \to \beta_i}$$

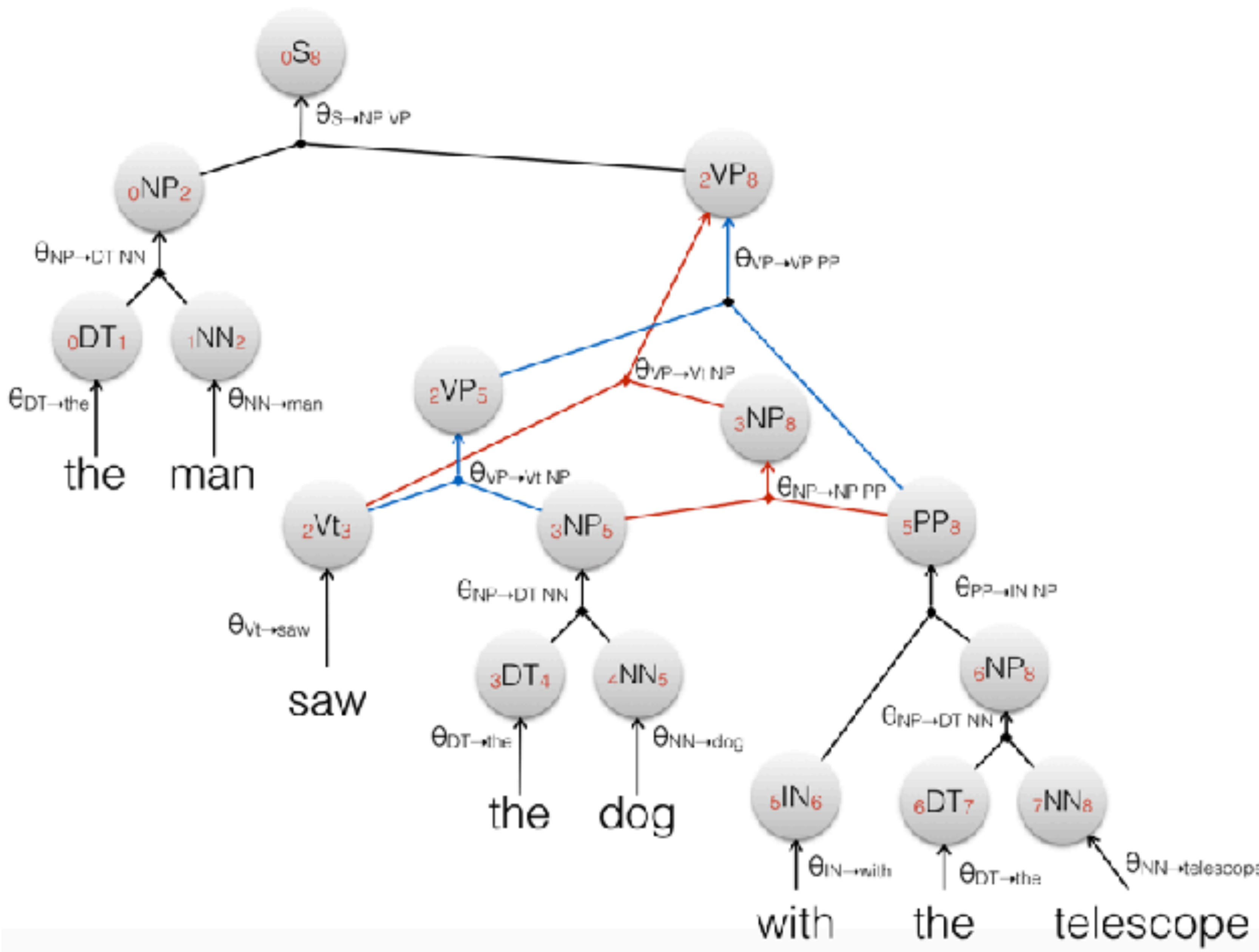
Marginal Probability



Marginal Probability

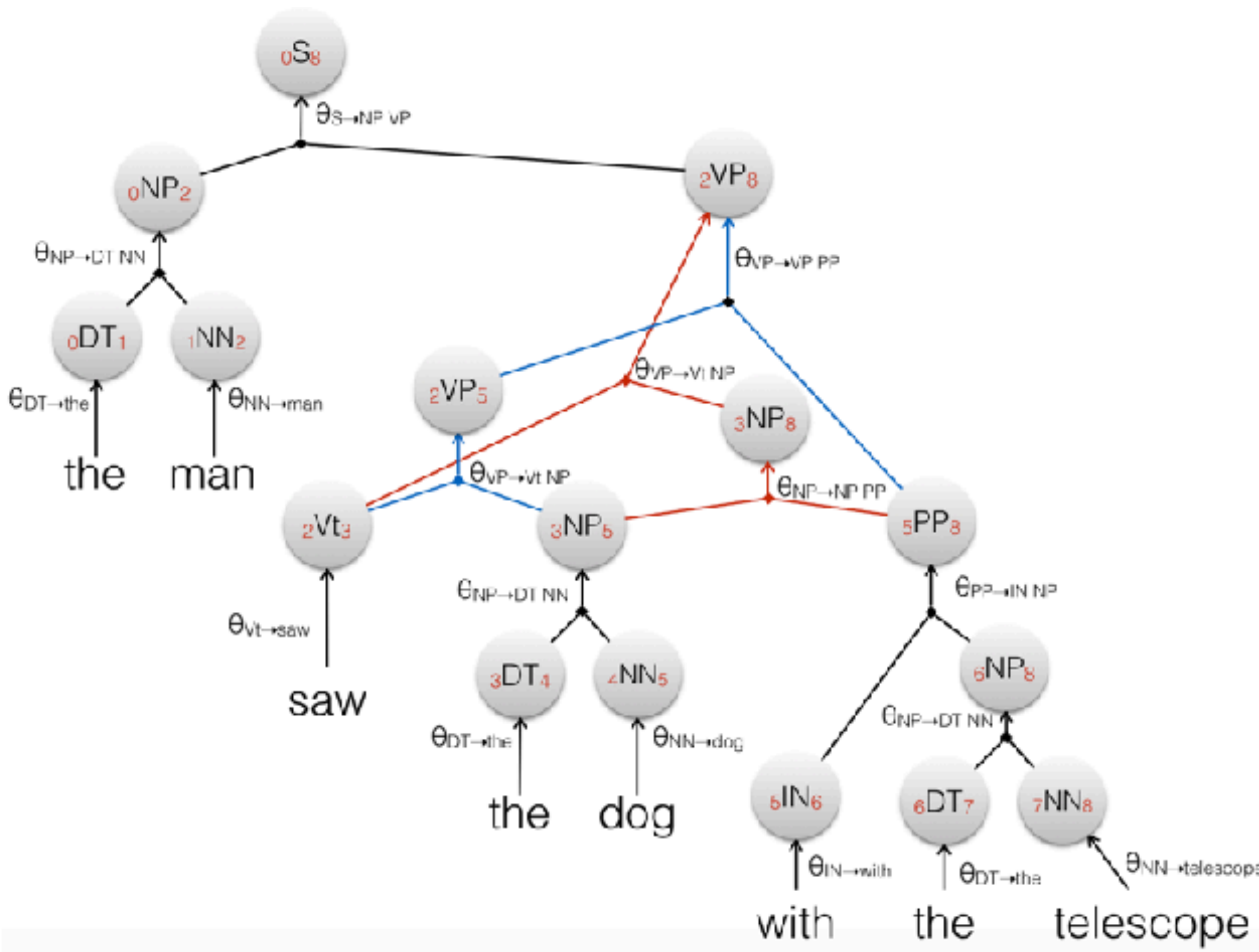


Marginal Probability



- $I({}_0S_8) =$

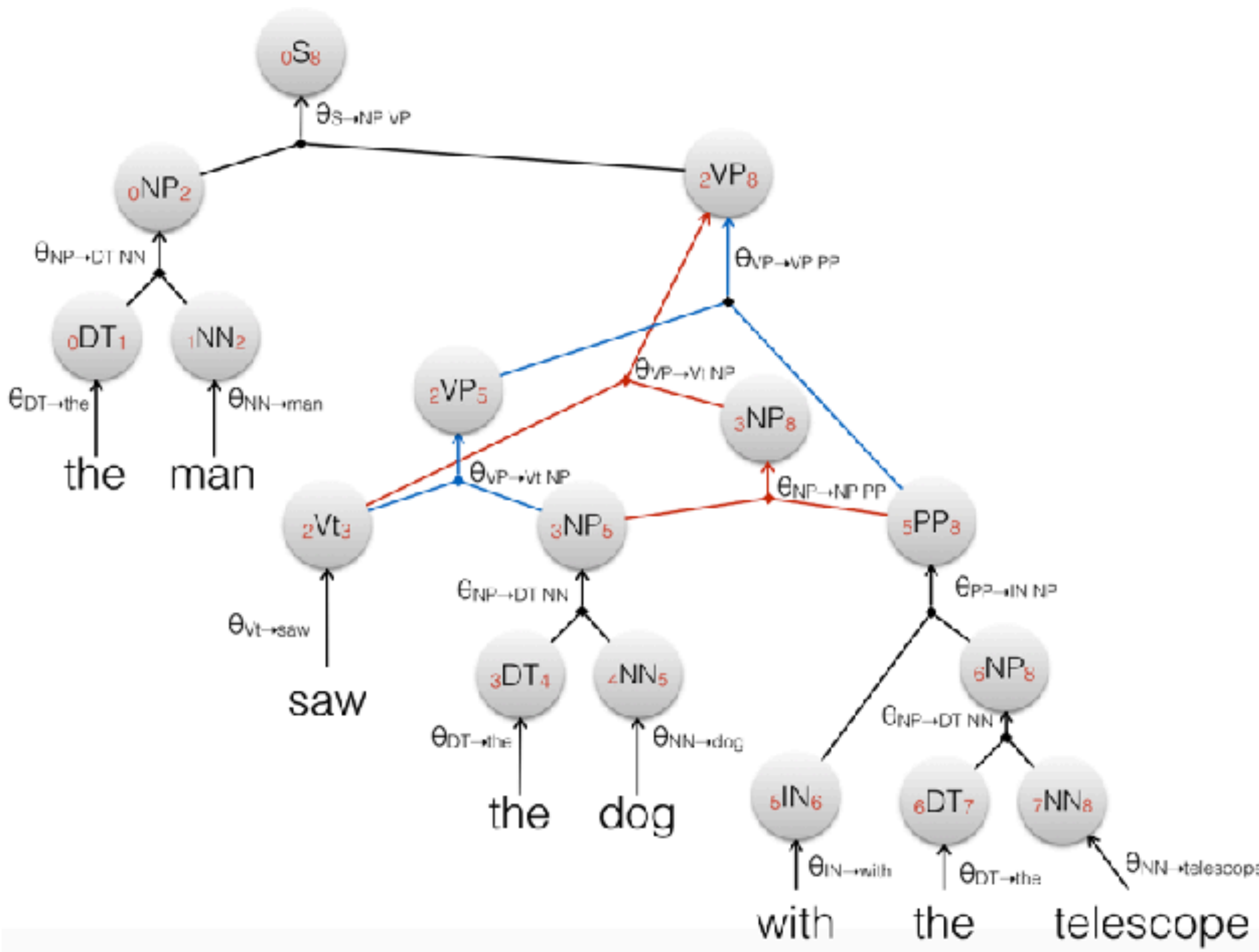
Marginal Probability



- $I({}_0S_8) =$

  $\theta_{S \to NP\ VP}\, I({}_0NP_2)\, I({}_2VP_8)$

Marginal Probability

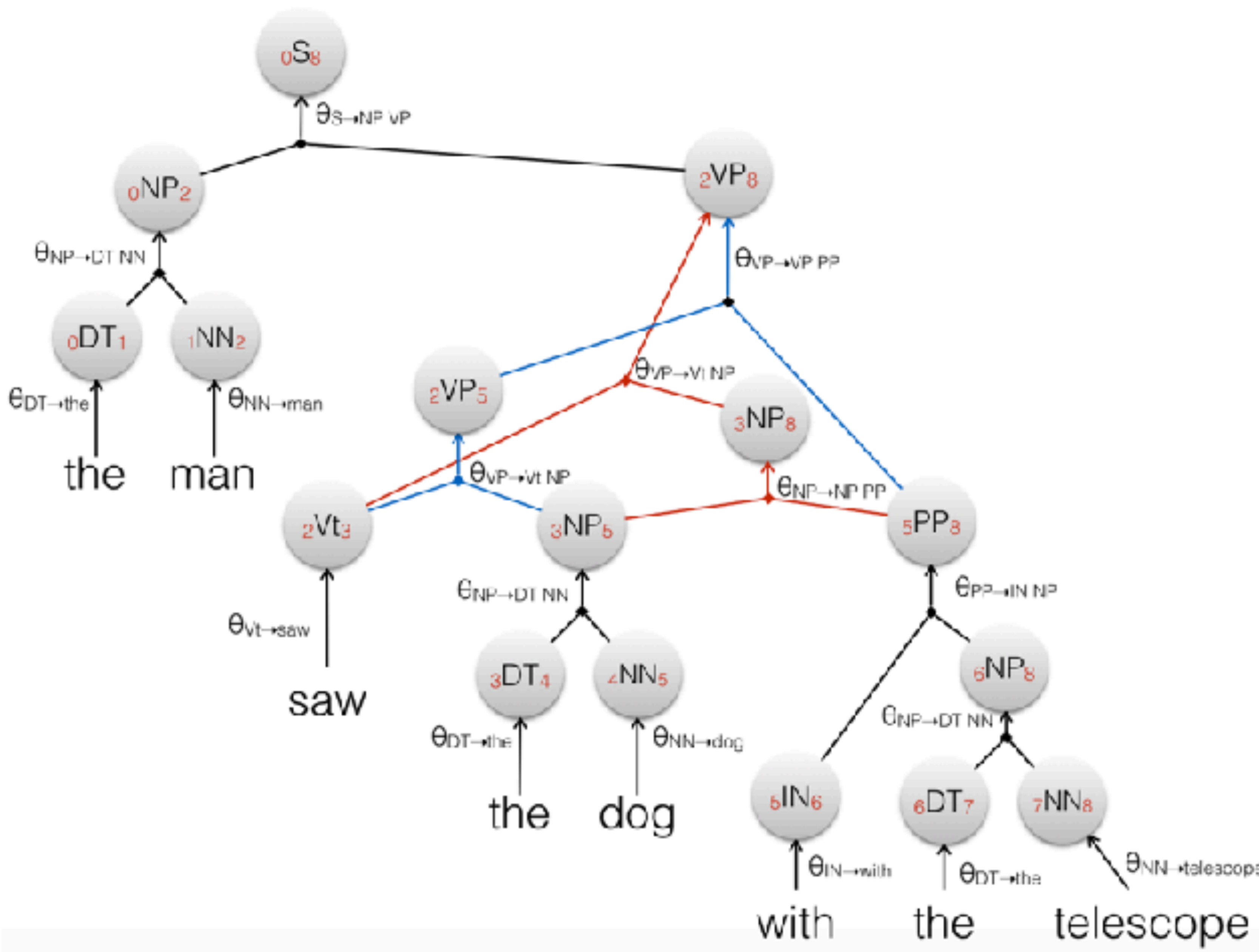


- $I({}_0S_8) =$

  $\theta_{S \to NP\ VP}\ I({}_0NP_2)\ I({}_2VP_8)$

- $I({}_0NP_2) =$

Marginal Probability



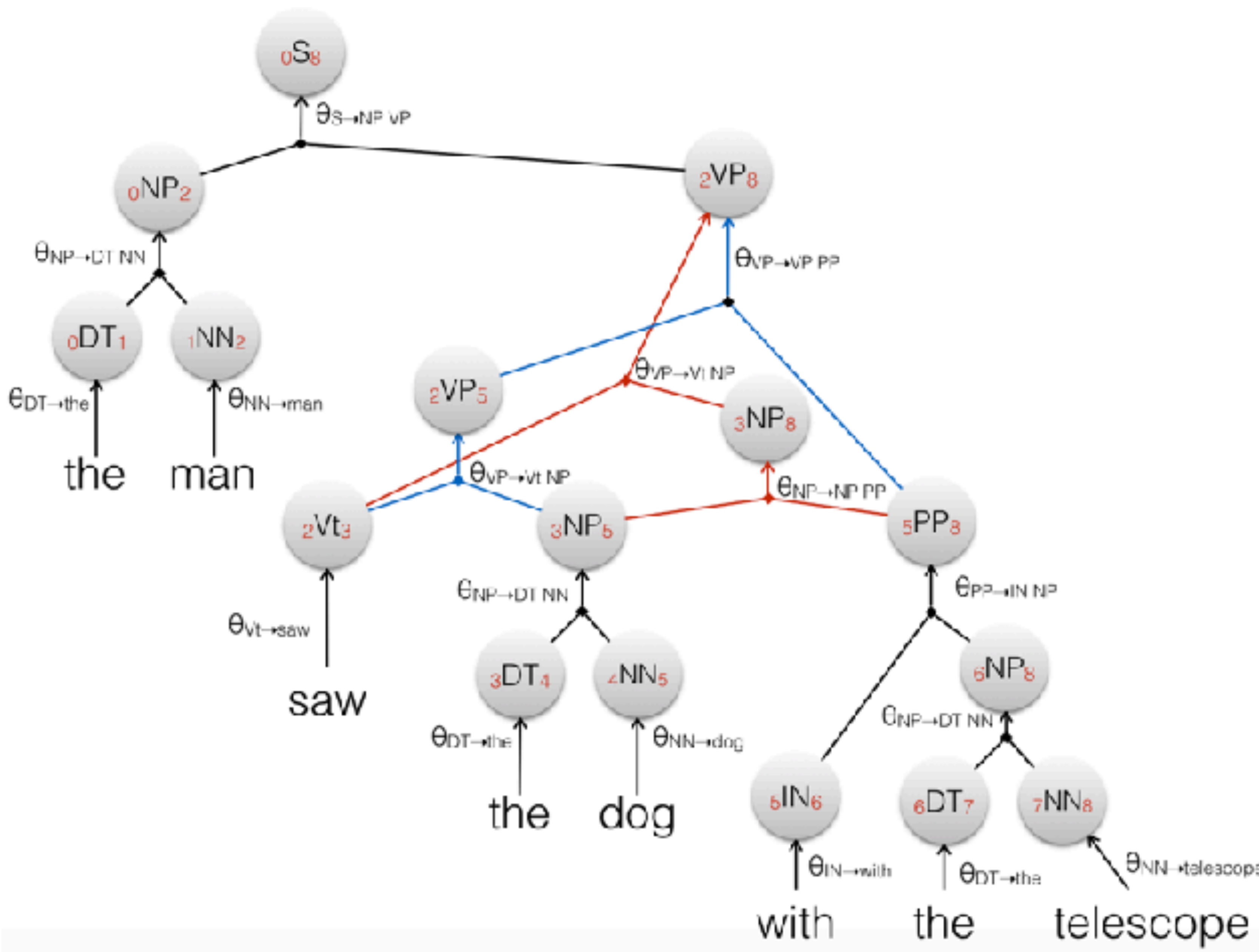
- $I({}_0S_8) =$

  $\theta_{S\rightarrow NP\ VP}\ I({}_0NP_2)\ I({}_2VP_8)$

- $I({}_0NP_2) =$

  $\theta_{NP\rightarrow DT\ NN}\ I({}_0DT_1)\ I({}_1NN_2)$

Marginal Probability



- $I({}_0S_8) =$

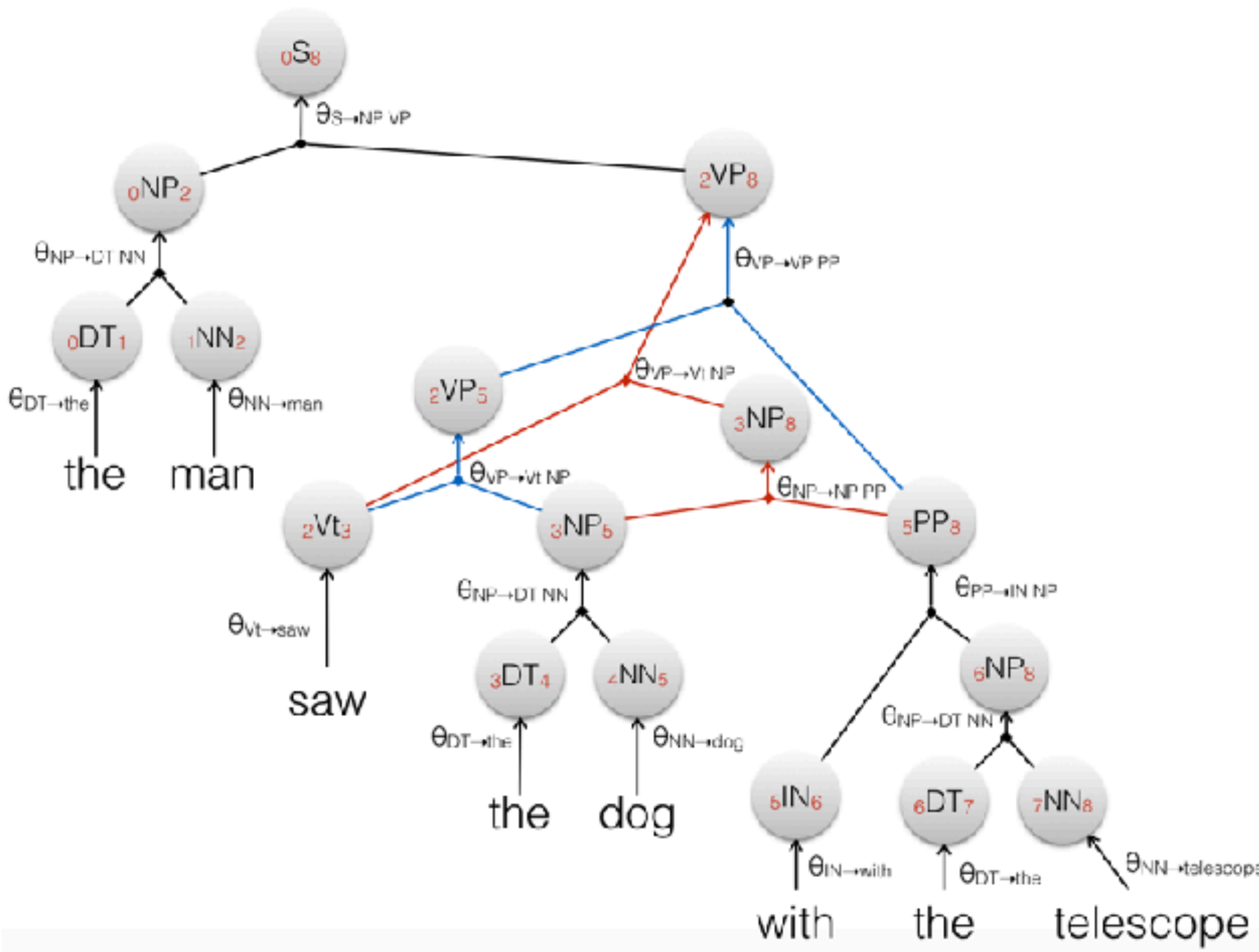
  $\theta_{S\to NP\ VP}\, I({}_0NP_2)\, I({}_2VP_8)$

- $I({}_0NP_2) =$

  $\theta_{NP\to DT\ NN}\, I({}_0DT_1)\, I({}_1NN_2)$

- $I({}_2VP_8) =$

Marginal Probability



- $I({}_0S_8) =$

  $\theta_{S\to NP\ VP}\ I({}_0NP_2)\ I({}_2VP_8)$

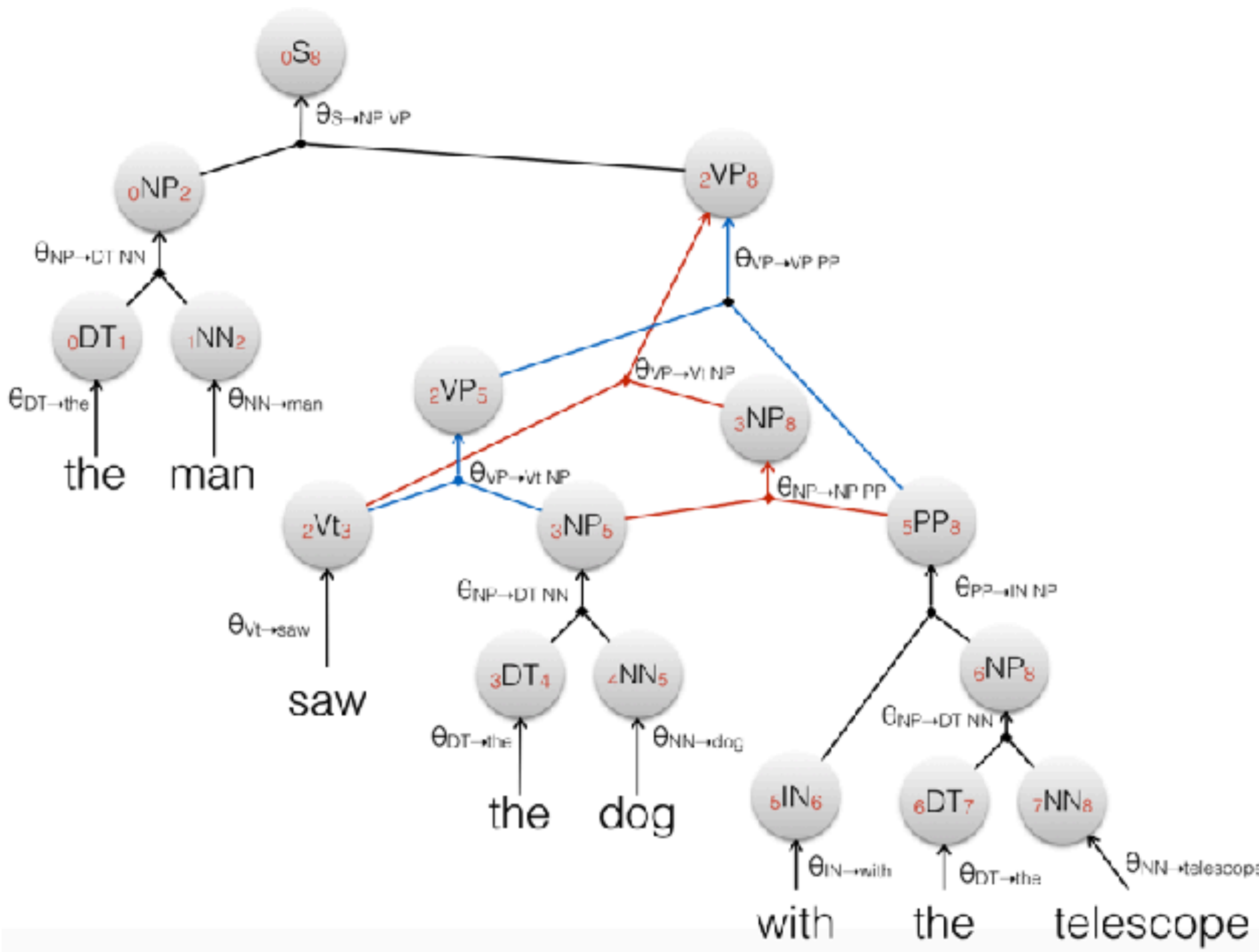
- $I({}_0NP_2) =$

  $\theta_{NP\to DT\ NN}\ I({}_0DT_1)\ I({}_1NN_2)$

- $I({}_2VP_8) =$

  $\theta_{VP\to VP\ PP}\ I({}_2VP_5)\ I({}_5PP_8)$

Marginal Probability



- $I({}_0S_8) =$

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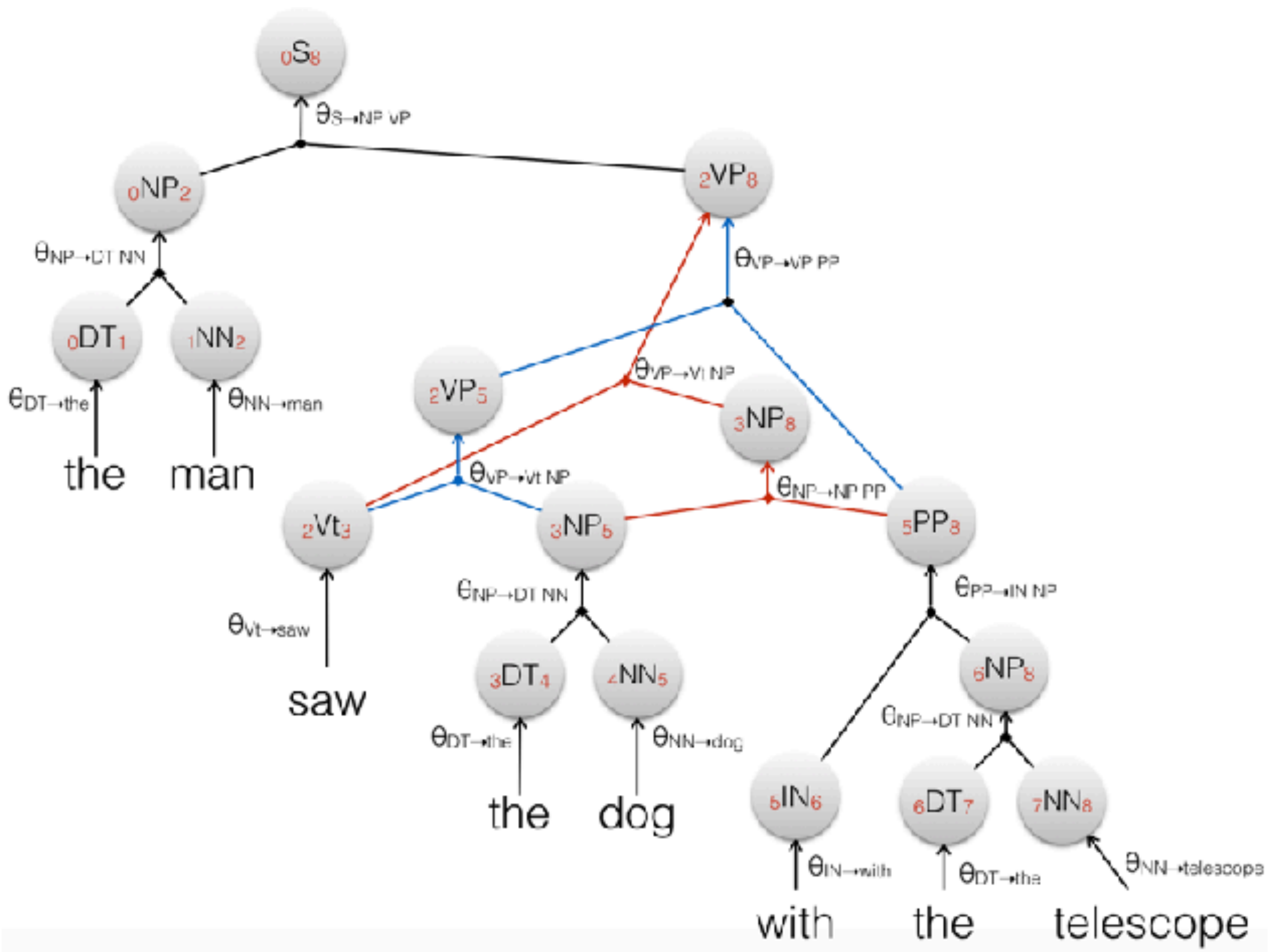

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…

Inside Weight

- Let us denote nodes/items by v, a_i
- Let us denote an edge/inference by $$\frac{a_1, \ldots, a_n}{v : \theta}$$
- θ is the weight of the rule underlying the inference
- $B(v)$ is the set of edges *incoming* to a node
  - i.e. *inferences* that prove the node

We call **Inside weight** the sum of weights of all derivations of a certain node

Inside recursion

$$I(v) = \begin{cases} 1 & \text{if } B(v) = \emptyset \\ \displaystyle\sum_{\frac{a_1, \ldots, a_n}{v:\theta} \in B(v)} \theta \times \prod_{i=1}^{n} I(a_i) & \text{otherwise} \end{cases}$$

For a PCFG, the **inside** of the GOAL node corresponds to the **marginal probability** of the sentence

$$P_{S|n}(x_1^n|n) = I({}_0\mathrm{S}_n) = \sum_{r_1^m \in \mathcal{G}(x_1^n)} \prod_{i=1}^{m} \theta_{v_i \to \beta_i}$$

Maximum Probability

$$\max_{r_1^m \in \mathcal{G}(x_1^n)} P_{ST|NM}(x_1^n, r_1^m | n, m) = V({}_0\mathrm{S}_n)$$

$$= \max_{r_1^m \in \mathcal{G}(x_1^n)} \prod_{i=1}^{m} \theta_{v_i \to \beta_i}$$

Maximum Probability

Let the goal item **stand** for the sentence. What's the probability of best tree under it V($_0$S$_8$)?

$$\max_{r_1^m \in \mathcal{G}(x_1^n)} P_{ST|NM}(x_1^n, r_1^m | n, m) = V({}_0\mathrm{S}_n)$$

$$= \max_{r_1^m \in \mathcal{G}(x_1^n)} \prod_{i=1}^{m} \theta_{v_i \to \beta_i}$$

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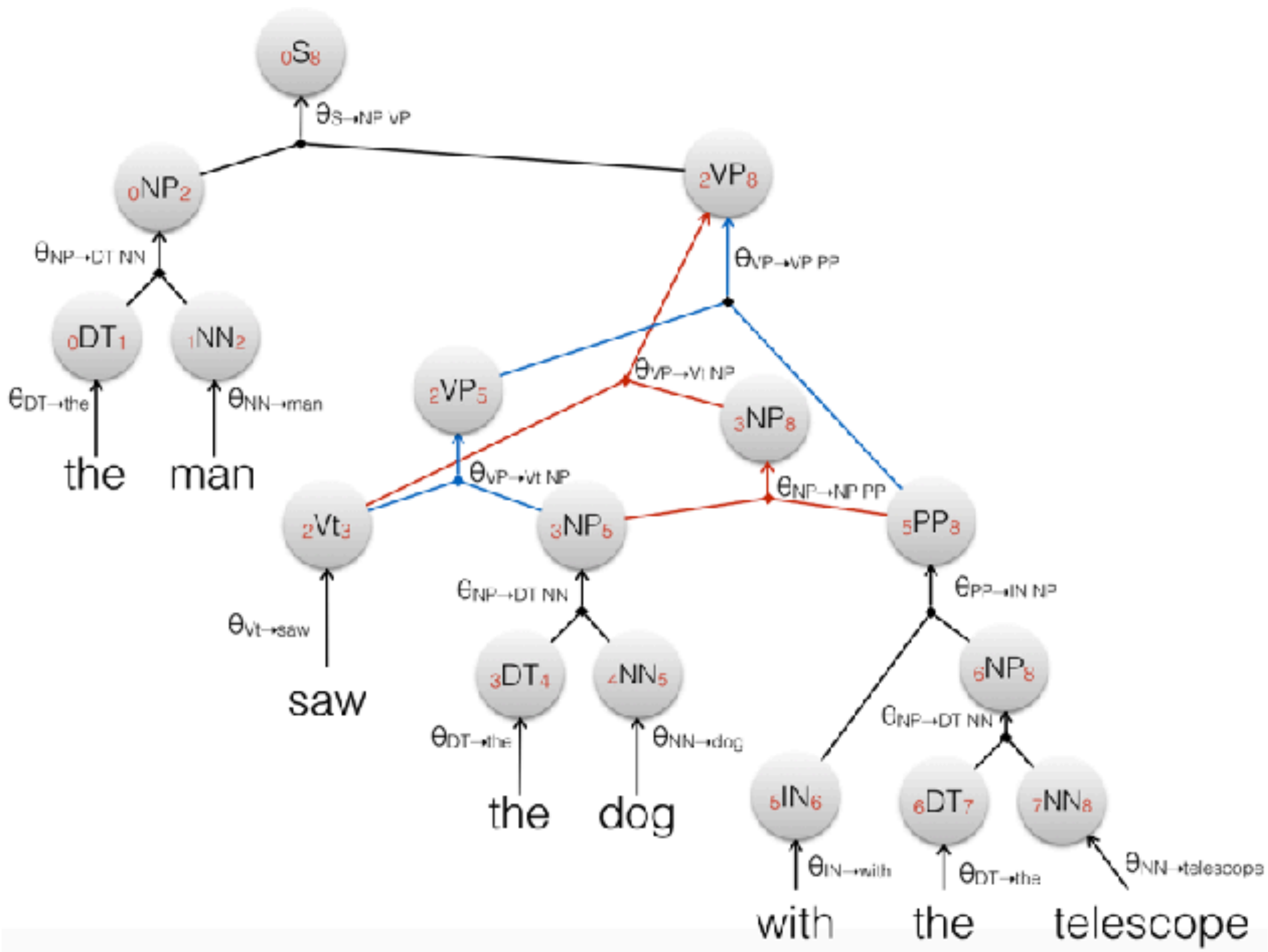

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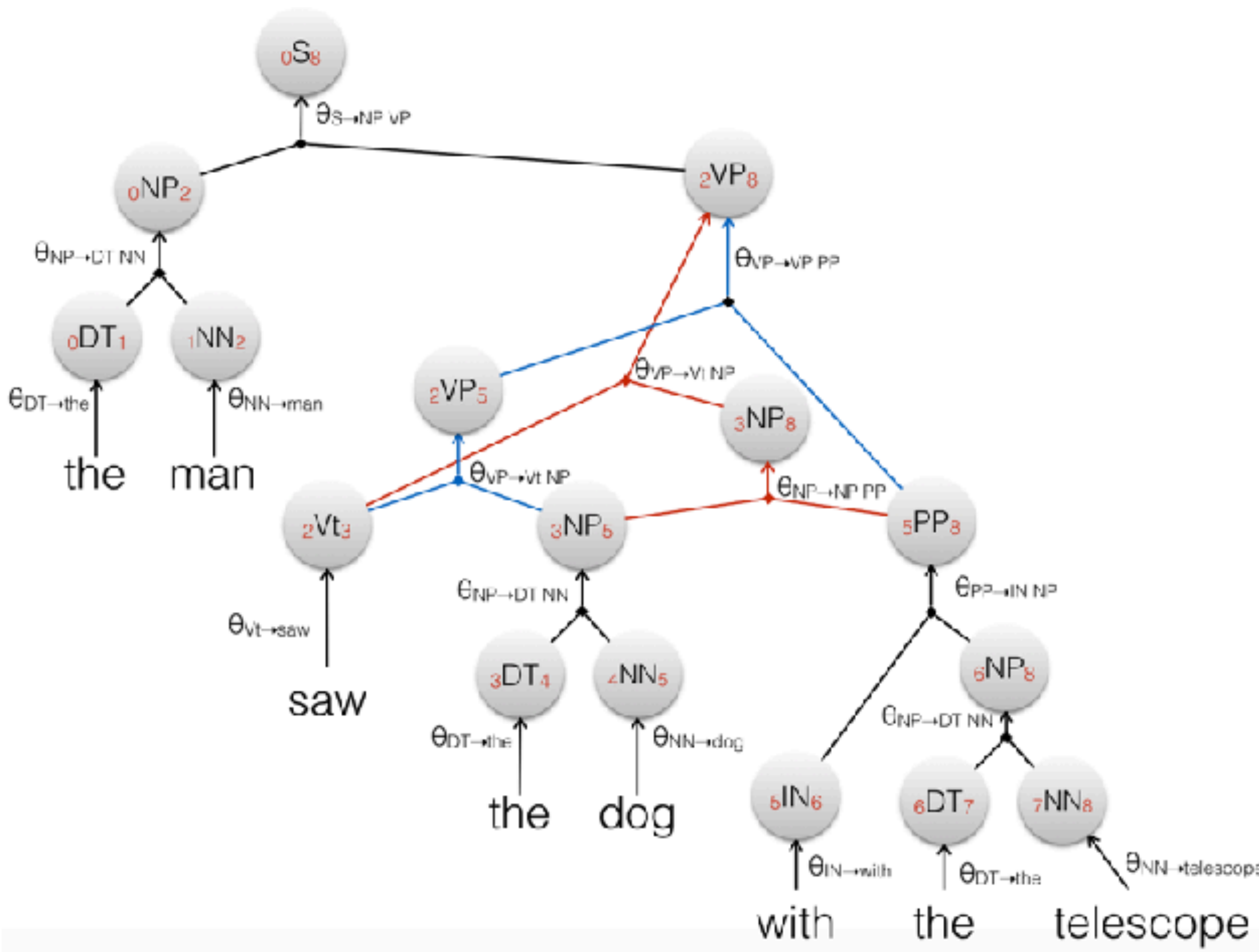

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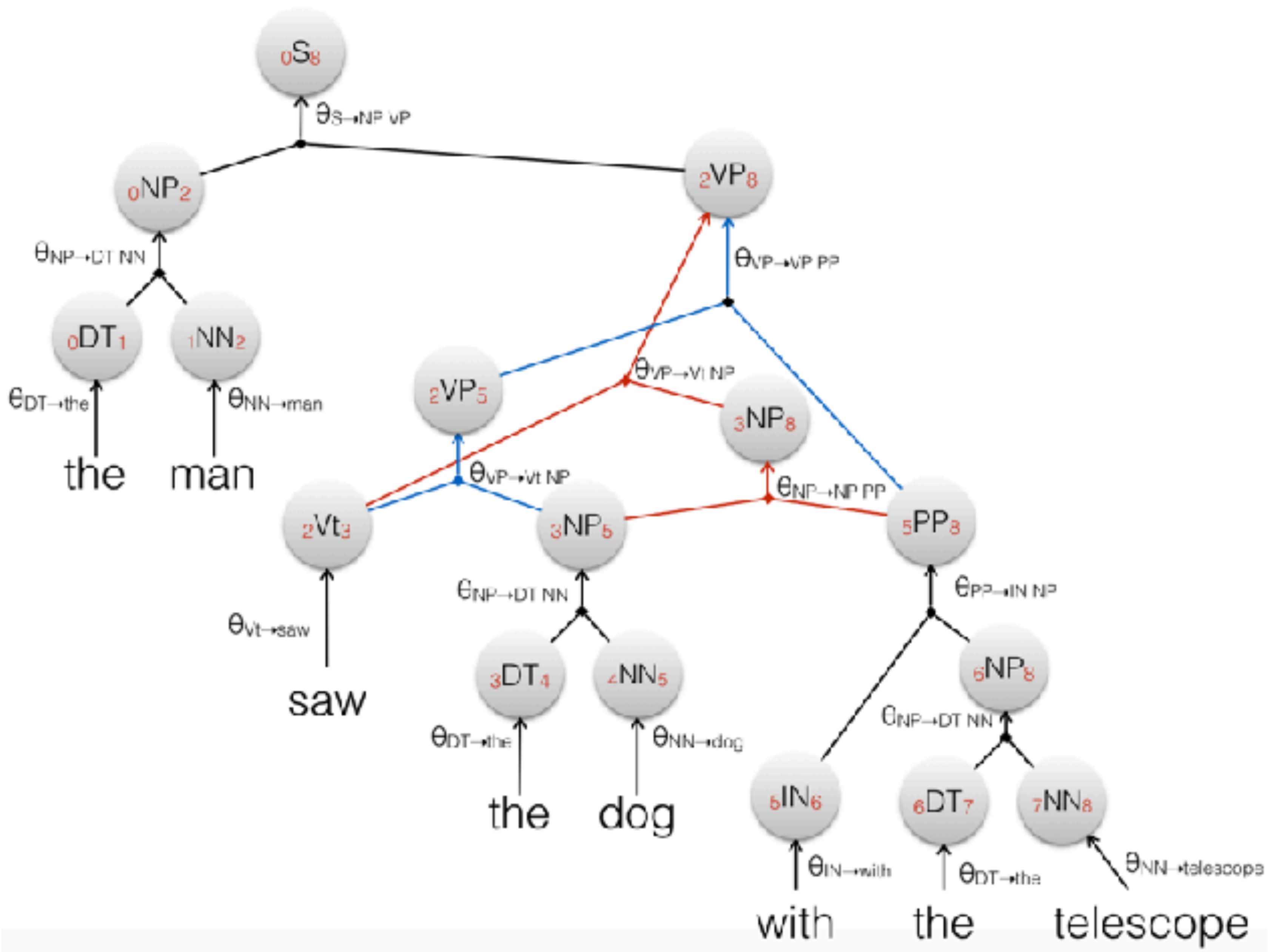
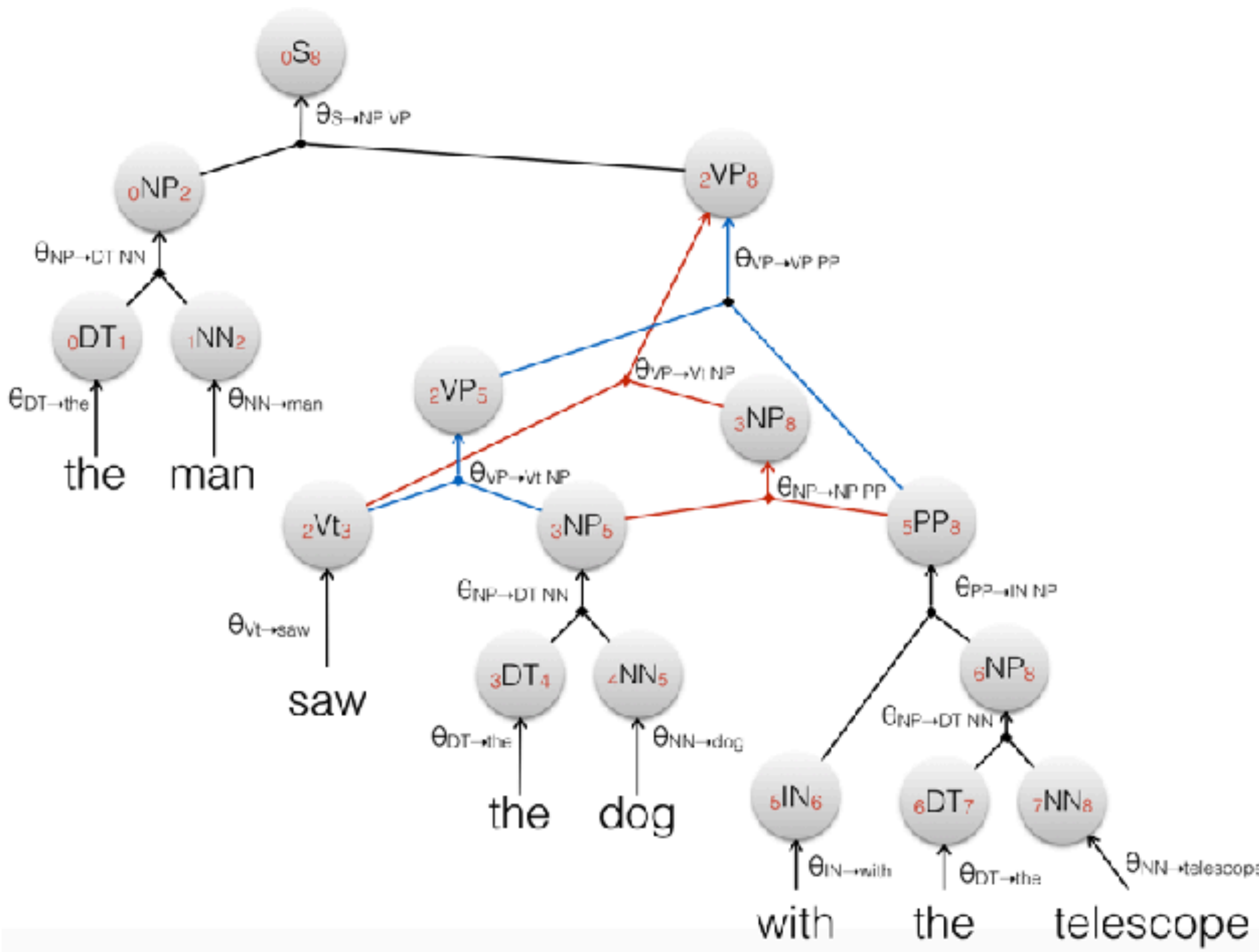


- $V({}_0S_8) =$

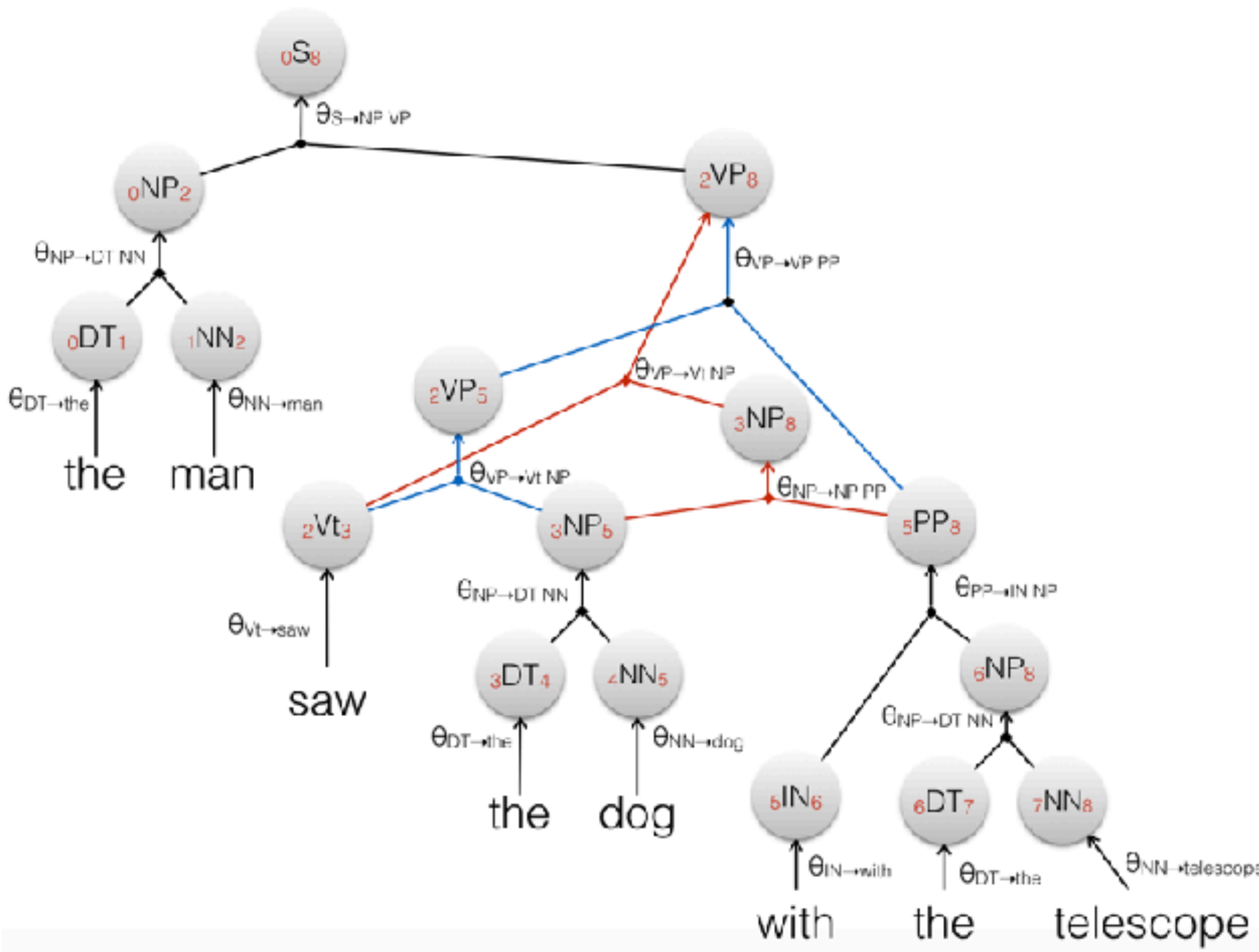
Maximum Probability



- $V({}_0S_8) =$

$\theta_{S\rightarrow NP\ VP}\ V({}_0NP_2)\ V({}_2VP_8)$

Maximum Probability

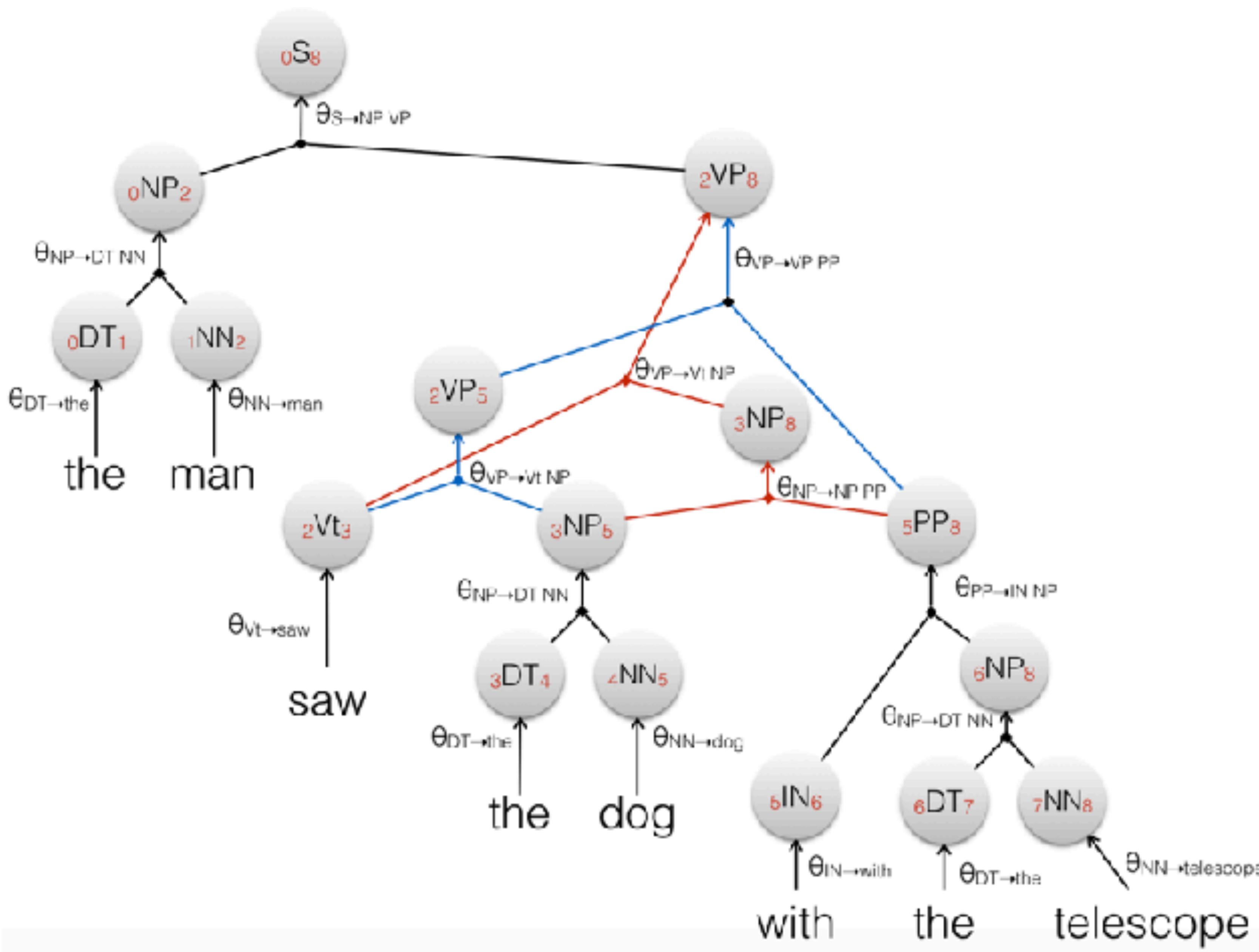


- $V({}_0S_8) =$

  $\theta_{S \to NP\ VP}\, V({}_0NP_2)\, V({}_2VP_8)$

- $V({}_0NP_2) =$

Maximum Probability



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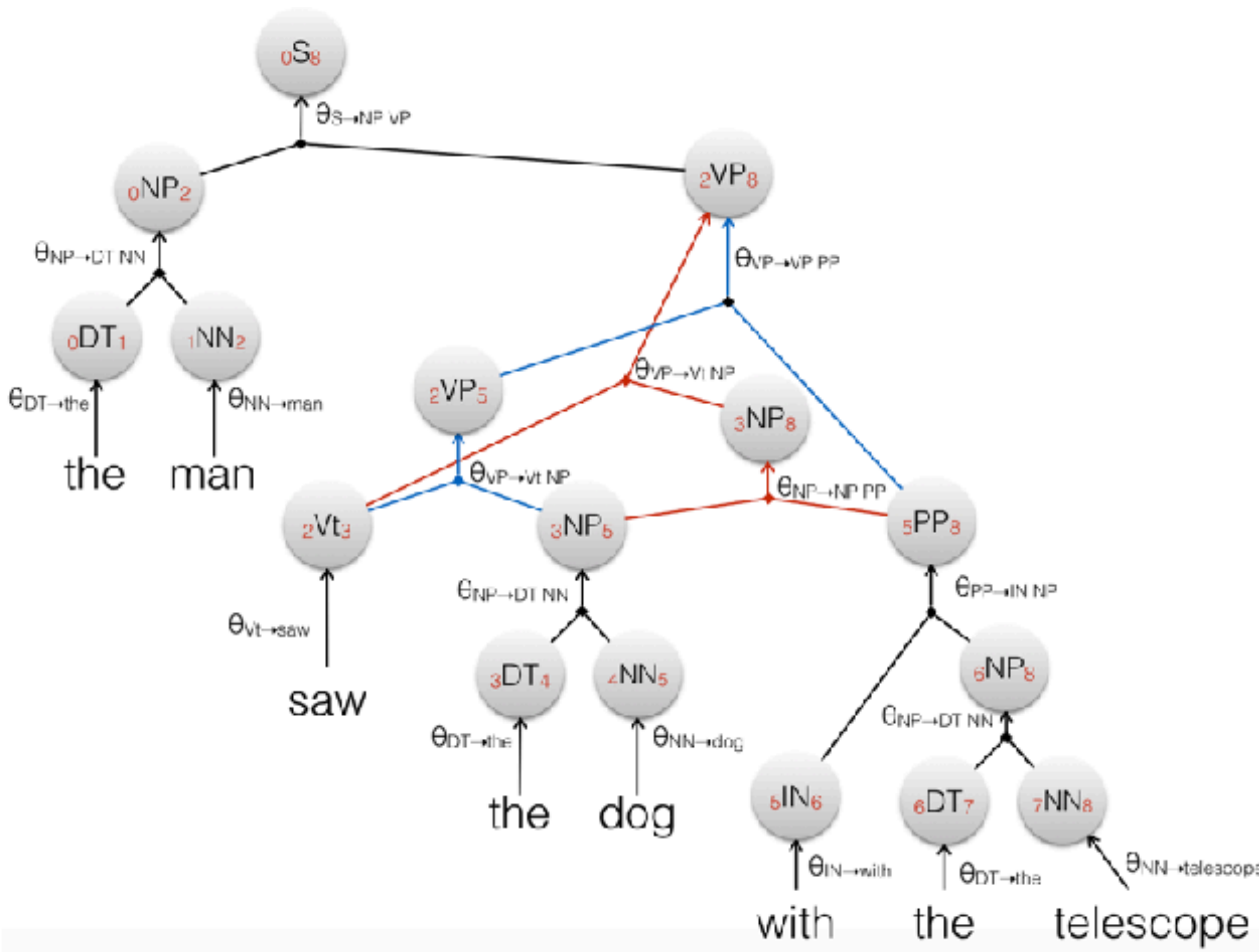
Maximum Probability



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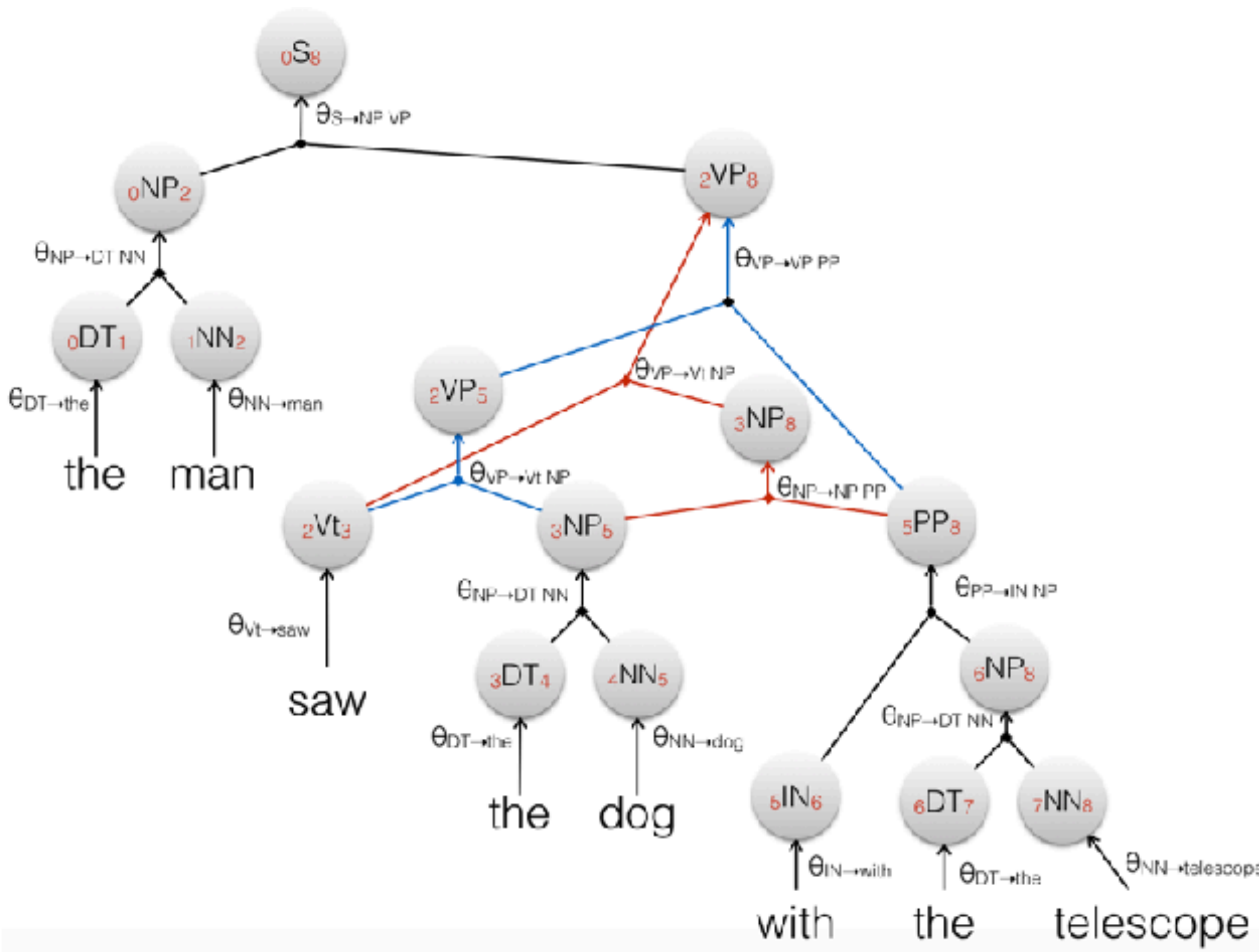

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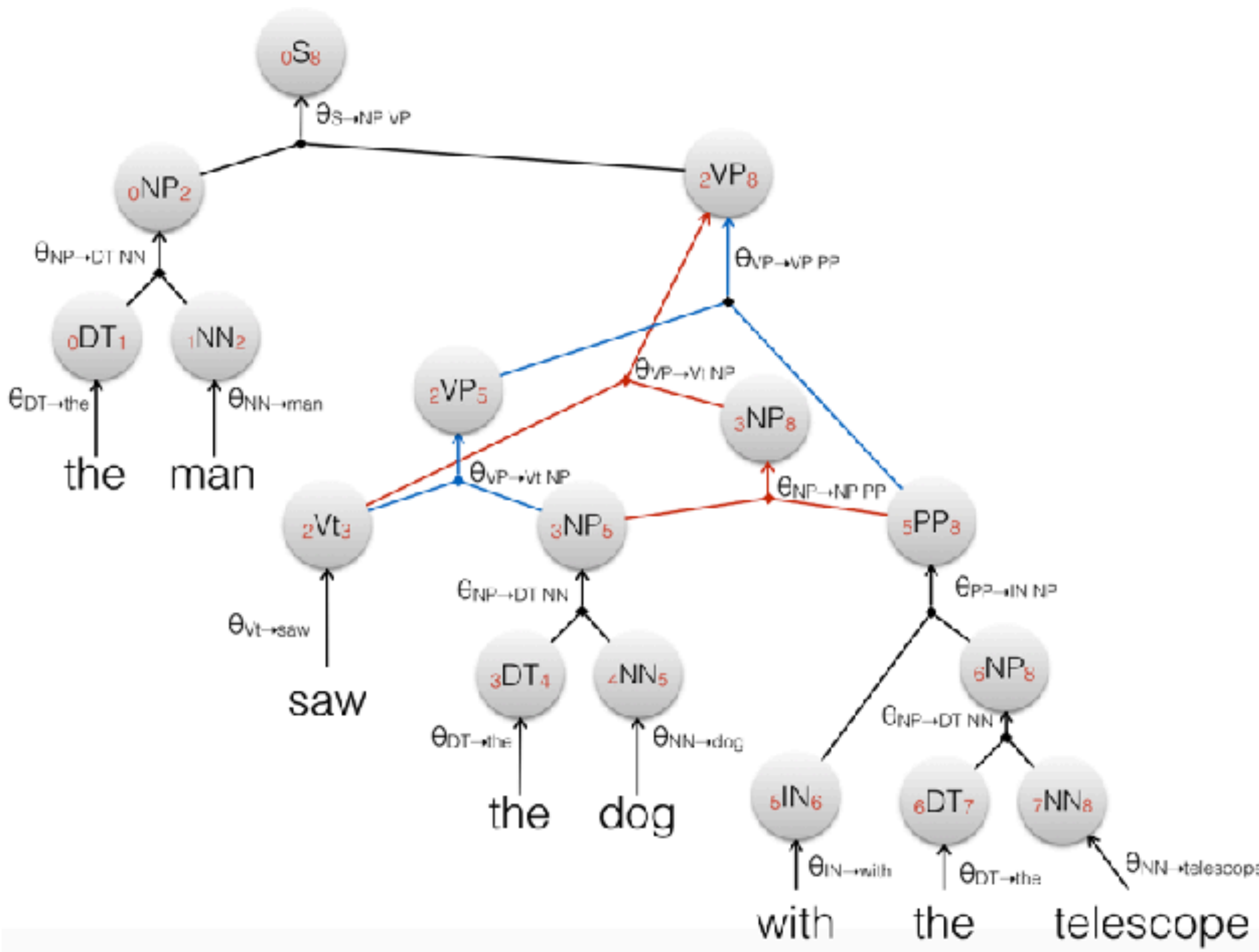
- $V({}_2VP_8) =$

  max {

Maximum Probability



- $V({}_{0}S_{8})$ =

  $\theta_{S \to NP\ VP}\ V({}_{0}NP_{2})\ V({}_{2}VP_{8})$

- $V({}_{0}NP_{2})$ =

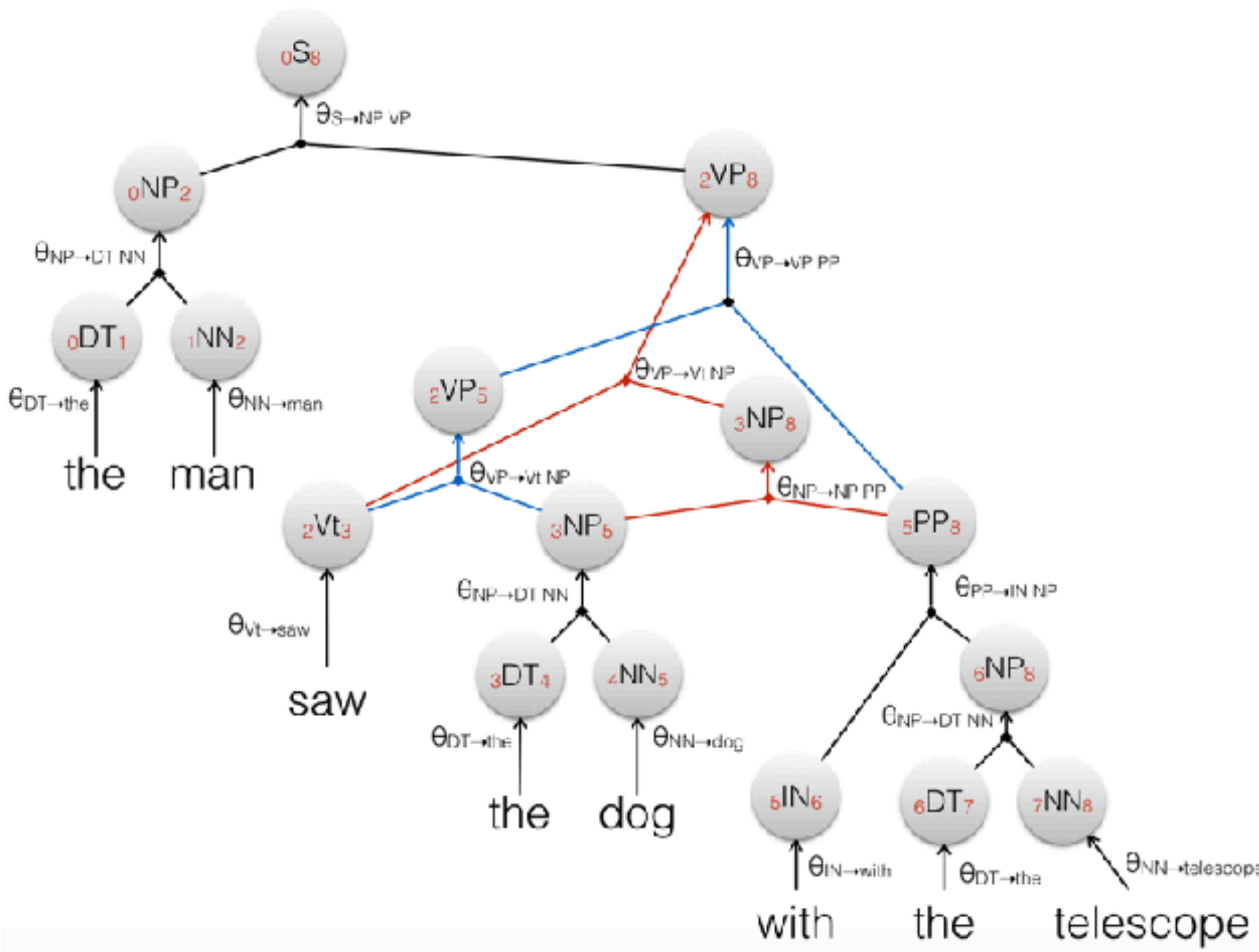
  $\theta_{NP \to DT\ NN}\ V({}_{0}DT_{1})\ V({}_{1}NN_{2})$

- $V({}_{2}VP_{8})$ =

  max {

  $\theta_{VP \to VP\ PP}\ V({}_{2}VP_{5})\ V({}_{5}PP_{8})$**,**

Maximum Probability



- $V({}_{0}S_{8})$ =

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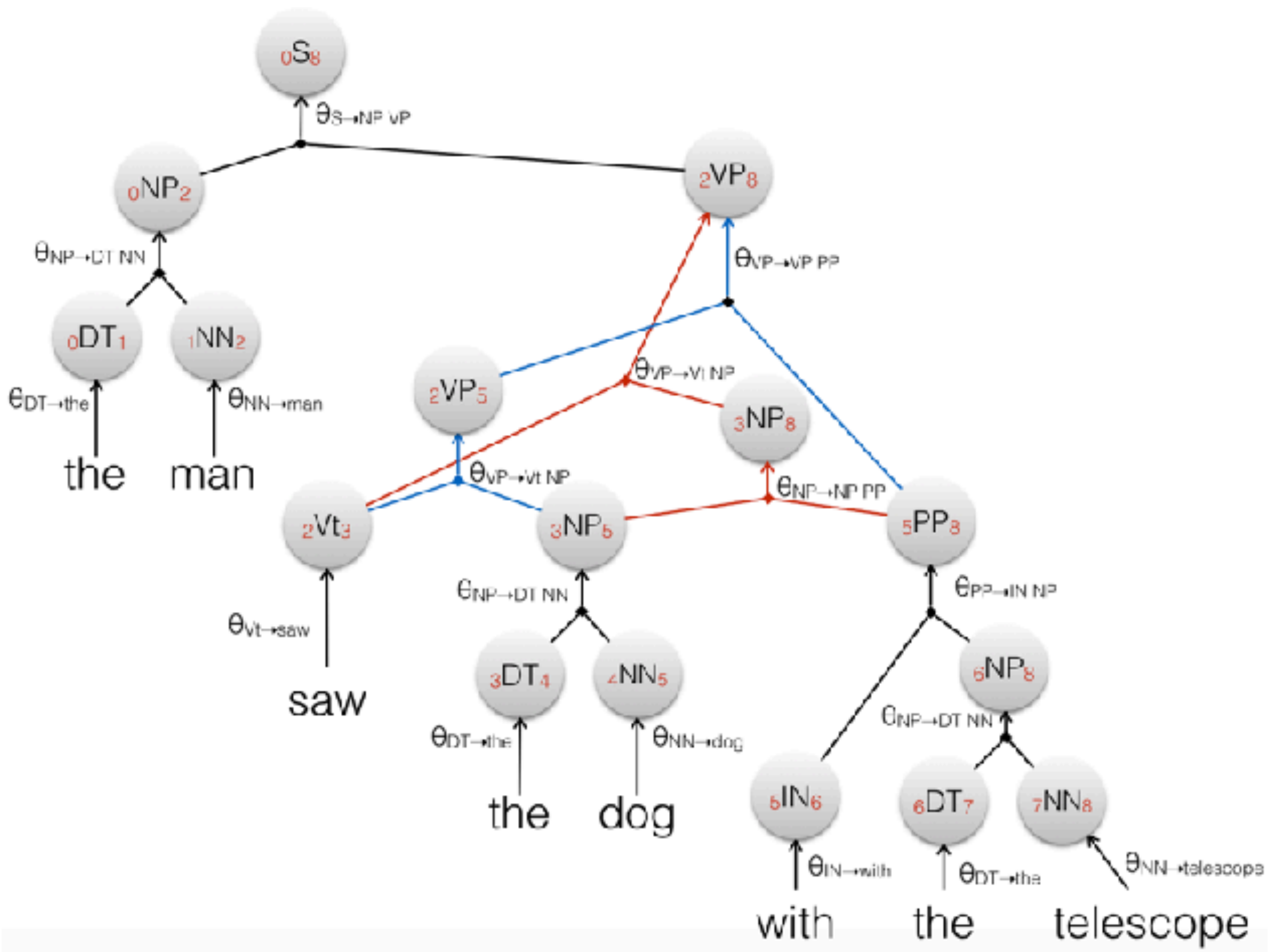
  $\theta_{VP\rightarrow VP\ PP}\ V({}_{2}VP_{5})\ V({}_{5}PP_{8})$**,**

  $\theta_{VP\rightarrow Vt\ NP}\ V({}_{2}Vt_{3})\ V({}_{3}NP_{8})$ **}**

Maximum Probability



…

- $V({}_0S_8) =$

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  $\theta_{VP \rightarrow VP\ PP}\ V({}_2VP_5)\ V({}_5PP_8)$**,**

  $\theta_{VP \rightarrow Vt\ NP}\ V({}_2Vt_3)\ V({}_3NP_8)$ **}**

Viterbi

$$I_{\max}(v) = \begin{cases} 1 & \text{if } B(v) = \emptyset \\ \max\limits_{\frac{a_1,\ldots,a_n}{v:\theta} \in B(v)} \theta \times \prod_{i=1}^{n} I(a_i) & \text{otherwise} \end{cases}$$

For a PCFG, the **inside algorithm** computed with **max** instead of sum corresponds to the **probability of the best derivation** of the sentence

$$V({}_0\mathrm{S}_n) = I_{\max}({}_0\mathrm{S}_n) = \max_{r_1^m \in \mathcal{G}(x_1^n)} P_{ST|NM}(x_1^n, r_1^m | n, m)$$

Many in One

The inside recursion is very general

- It includes other dynamic programs
  - e.g. Viterbi

Semirings

- Generalise sum and products

Semirings

Marginal (probability)

$$a \oplus b = a + b$$
$$a \otimes b = a \times b$$
$$\bar{1} = 1$$
$$\bar{0} = 0$$

Log-marginal (probability)

$$a \oplus b = \log(\exp a + \exp b)$$
$$a \otimes b = a + b$$
$$\bar{1} = 0$$
$$\bar{0} = -\infty$$

Viterbi (max-probability)

$$a \oplus b = \max(a, b)$$
$$a \otimes b = a \times b$$
$$\bar{1} = 1$$
$$\bar{0} = 0$$

Log-viterbi (max-log-prob)

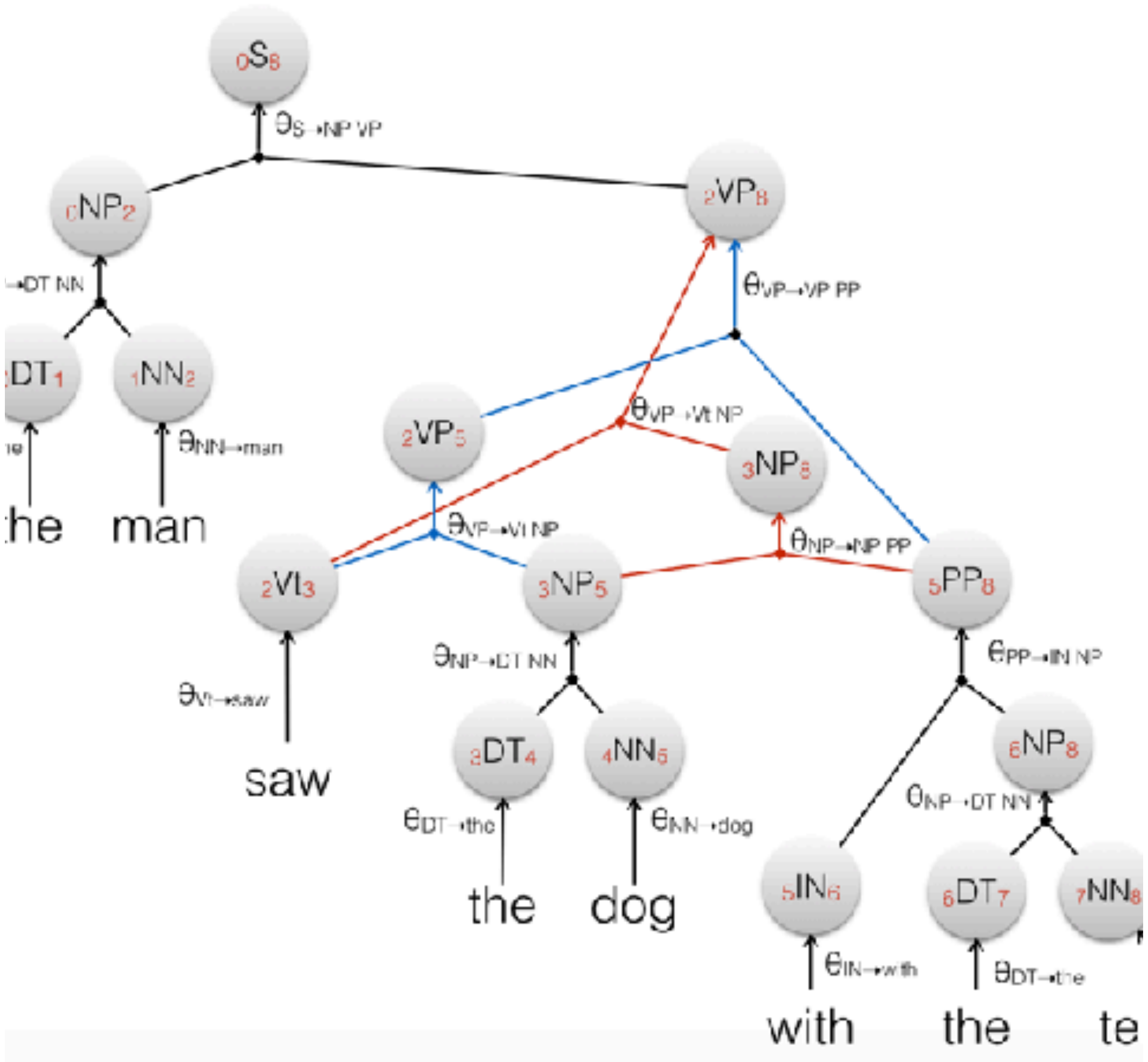
$$a \oplus b = \max(a, b)$$
$$a \otimes b = a + b$$
$$\bar{1} = 0$$
$$\bar{0} = -\infty$$

Inside semiring

With generalised operations

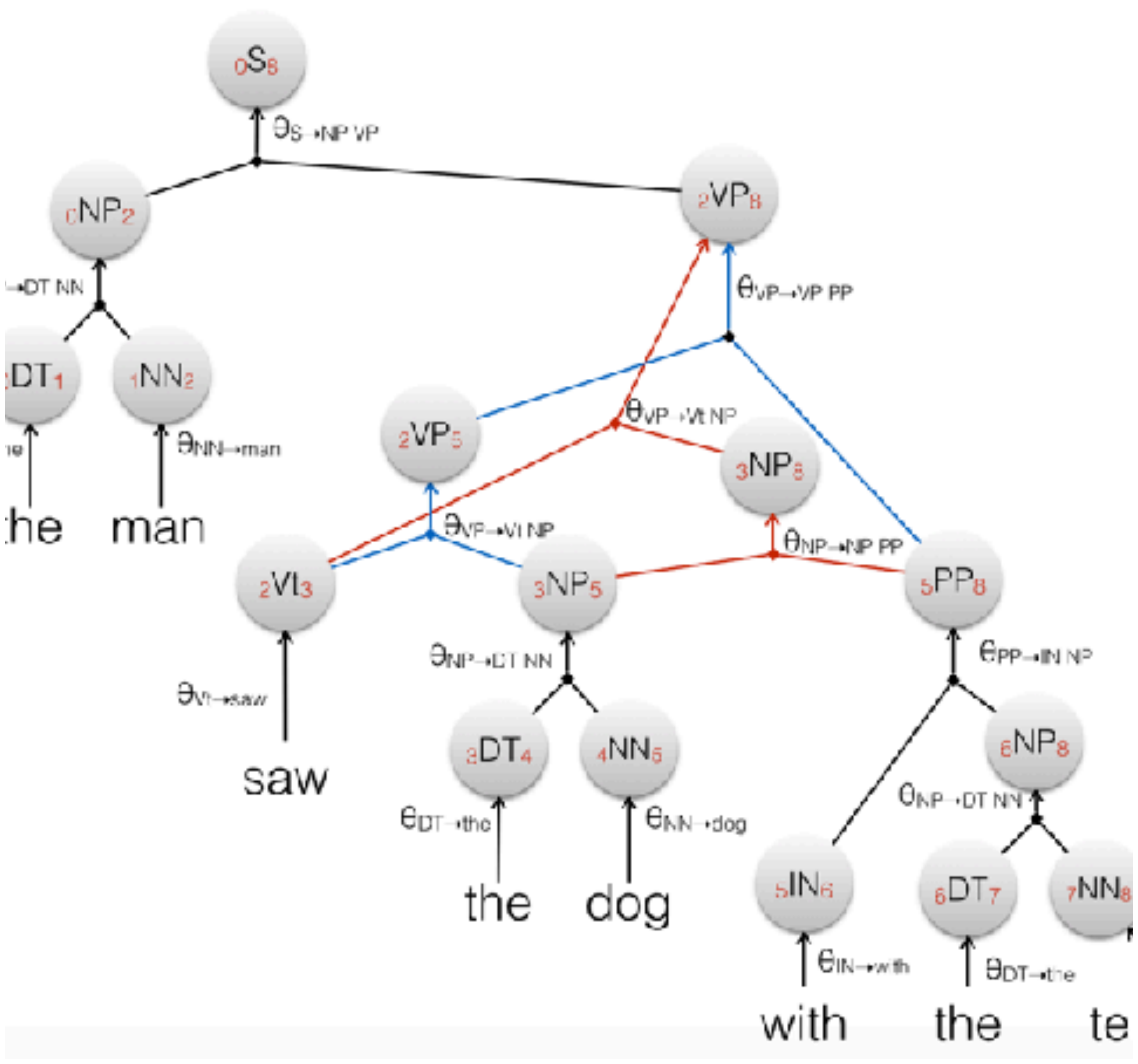
$$I(v) = \begin{cases} \bar{1} & \text{if } B(v) = \emptyset \\ \displaystyle\bigoplus_{\frac{a_1,\ldots,a_n}{v:\theta} \in B(v)} \theta \otimes \bigotimes_{i=1}^{n} I(a_i) & \text{otherwise} \end{cases}$$

Inside example



Inside example

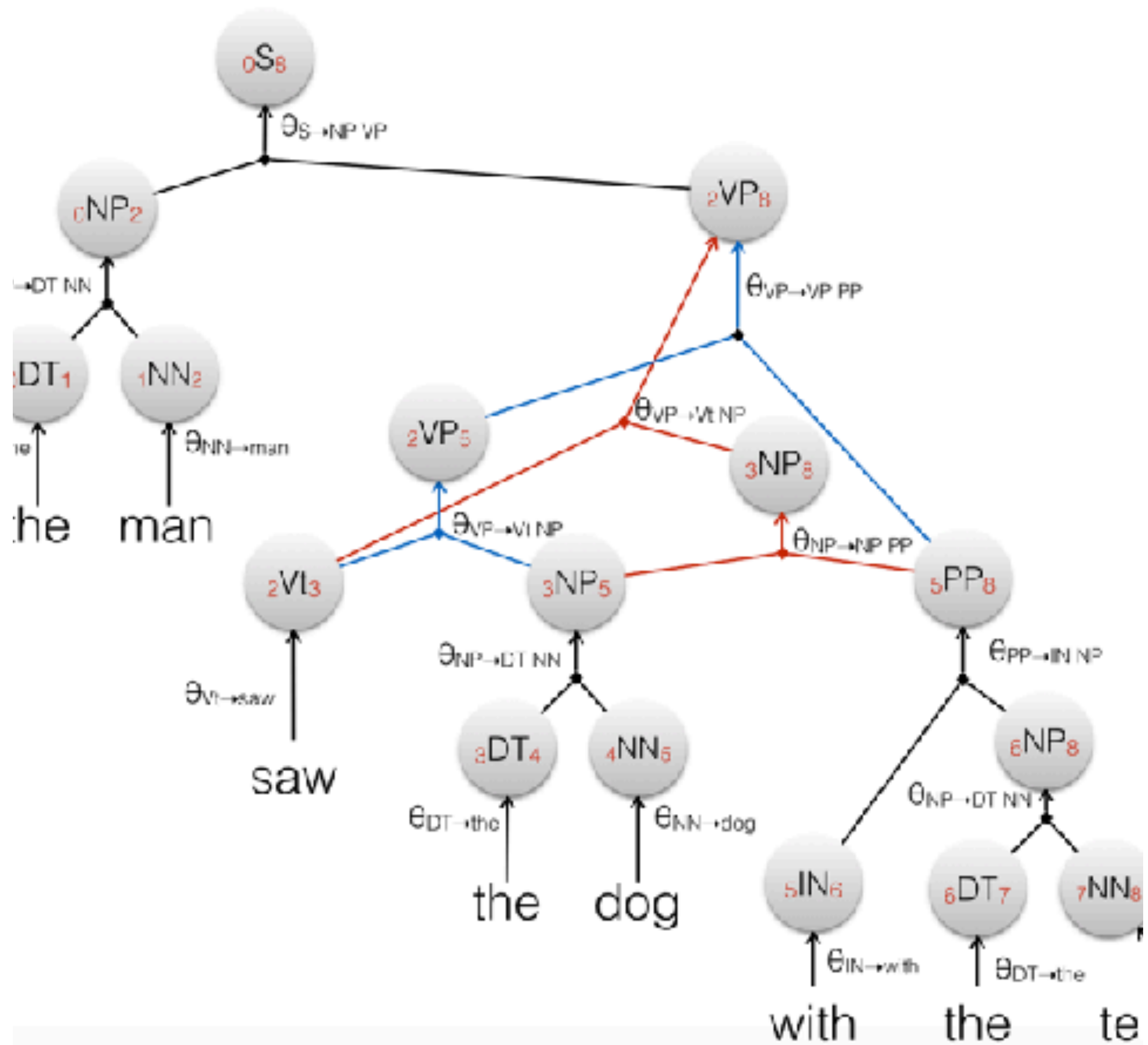
- $I({}_0S_8) =$



Inside example

- $I({}_0S_8) =$

  $\theta_{S\rightarrow NP\ VP} \otimes I({}_0NP_2) \otimes I({}_2VP_8)$



Inside example

- $I({}_0S_8) =$

  $\theta_{S\to NP\ VP} \otimes I({}_0NP_2) \otimes I({}_2VP_8)$

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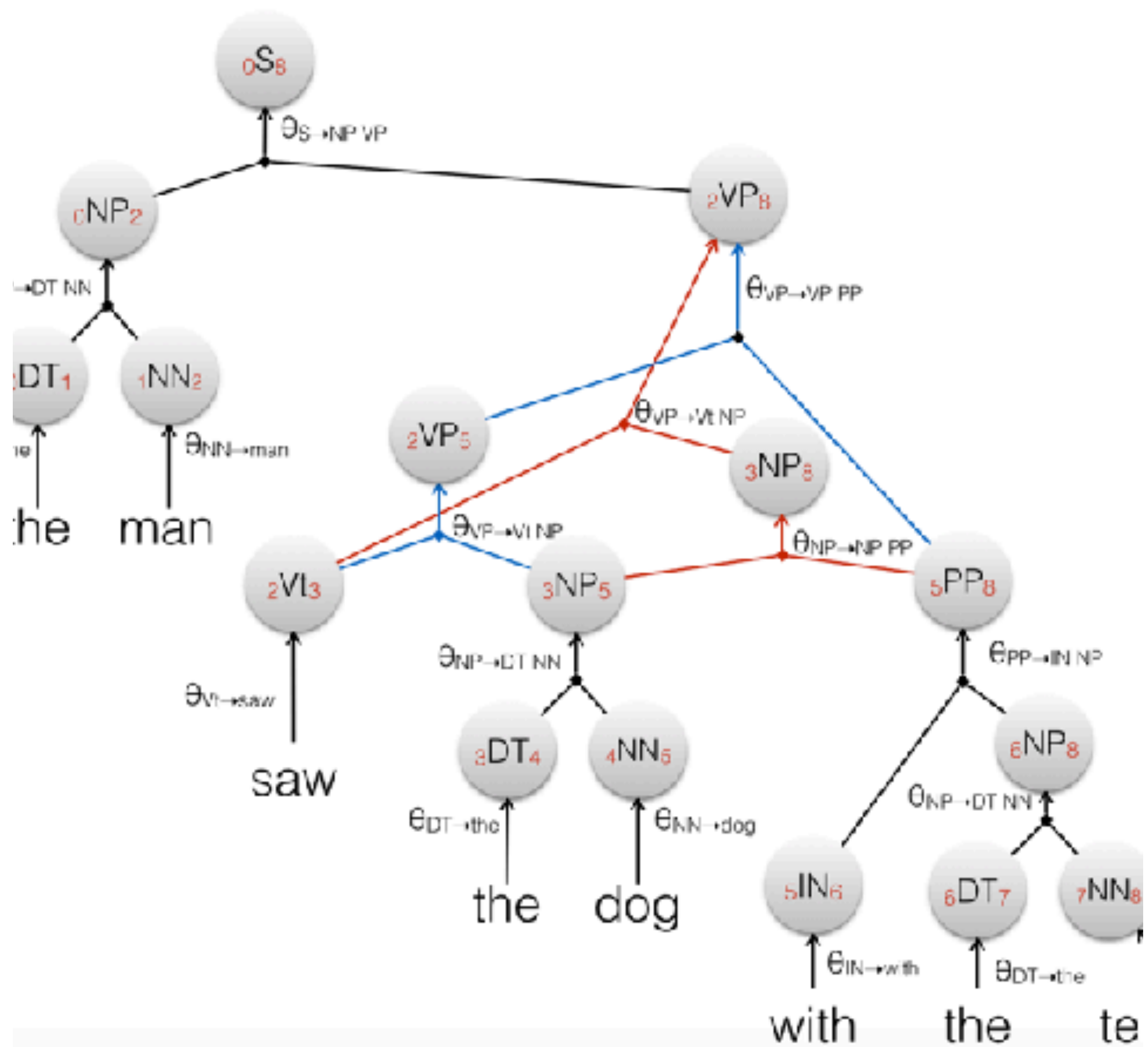

Inside example

- $I({}_0S_8)$ =

  $\theta_{S\to NP\ VP} \otimes I({}_0NP_2) \otimes I({}_2VP_8)$

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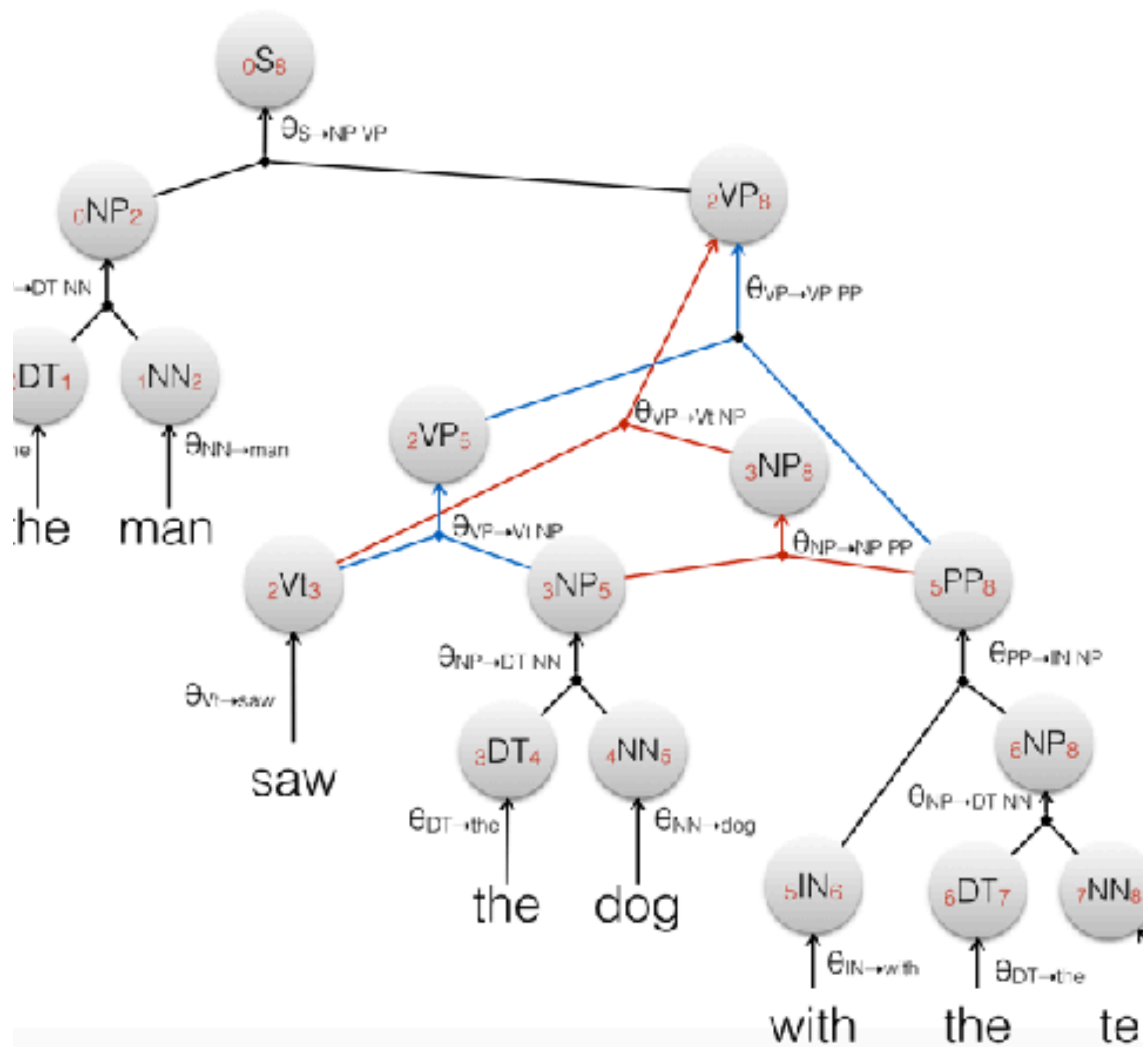

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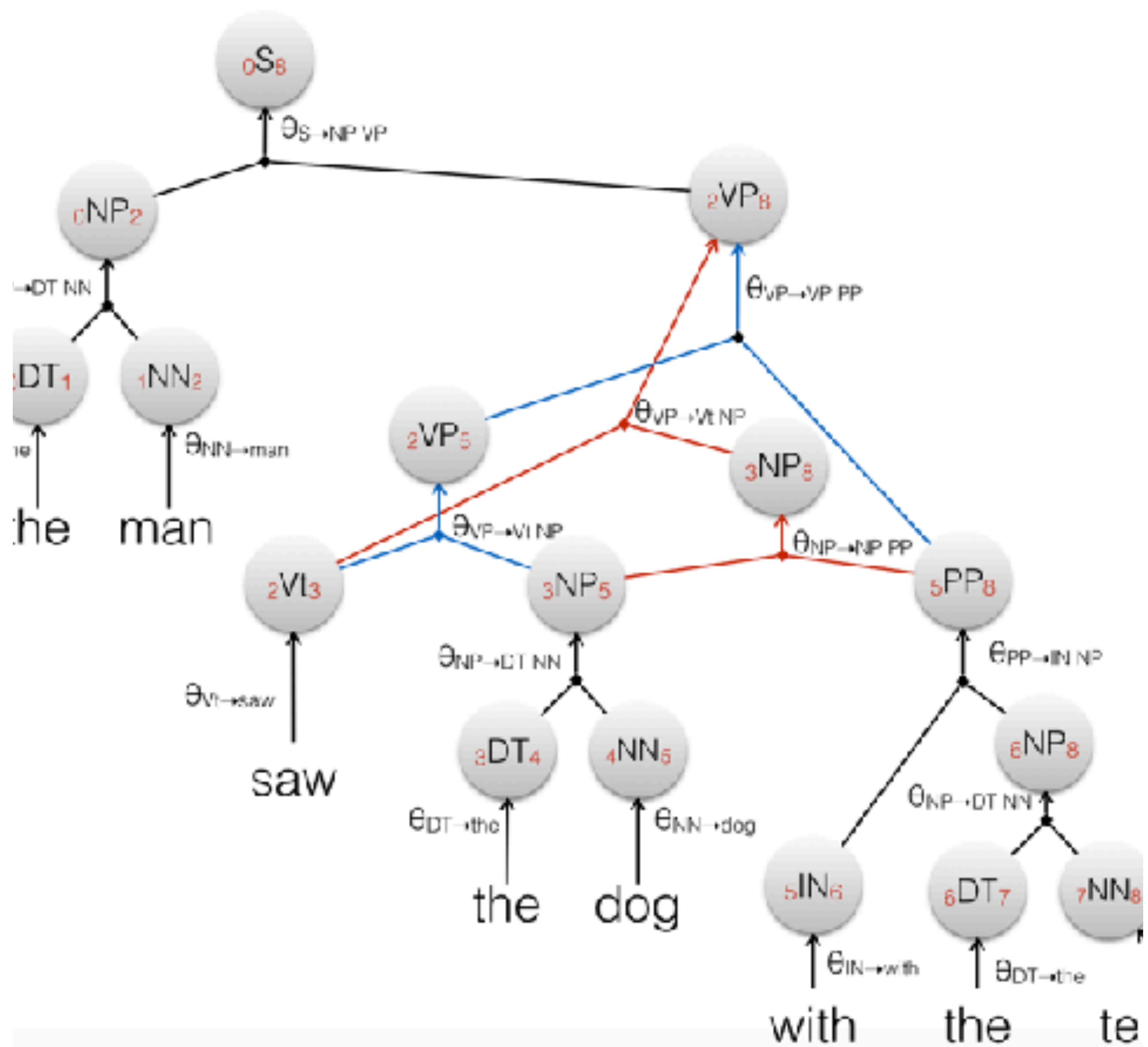
- $I({}_0DT_1) =$



Inside example

- $I({}_0S_8) =$

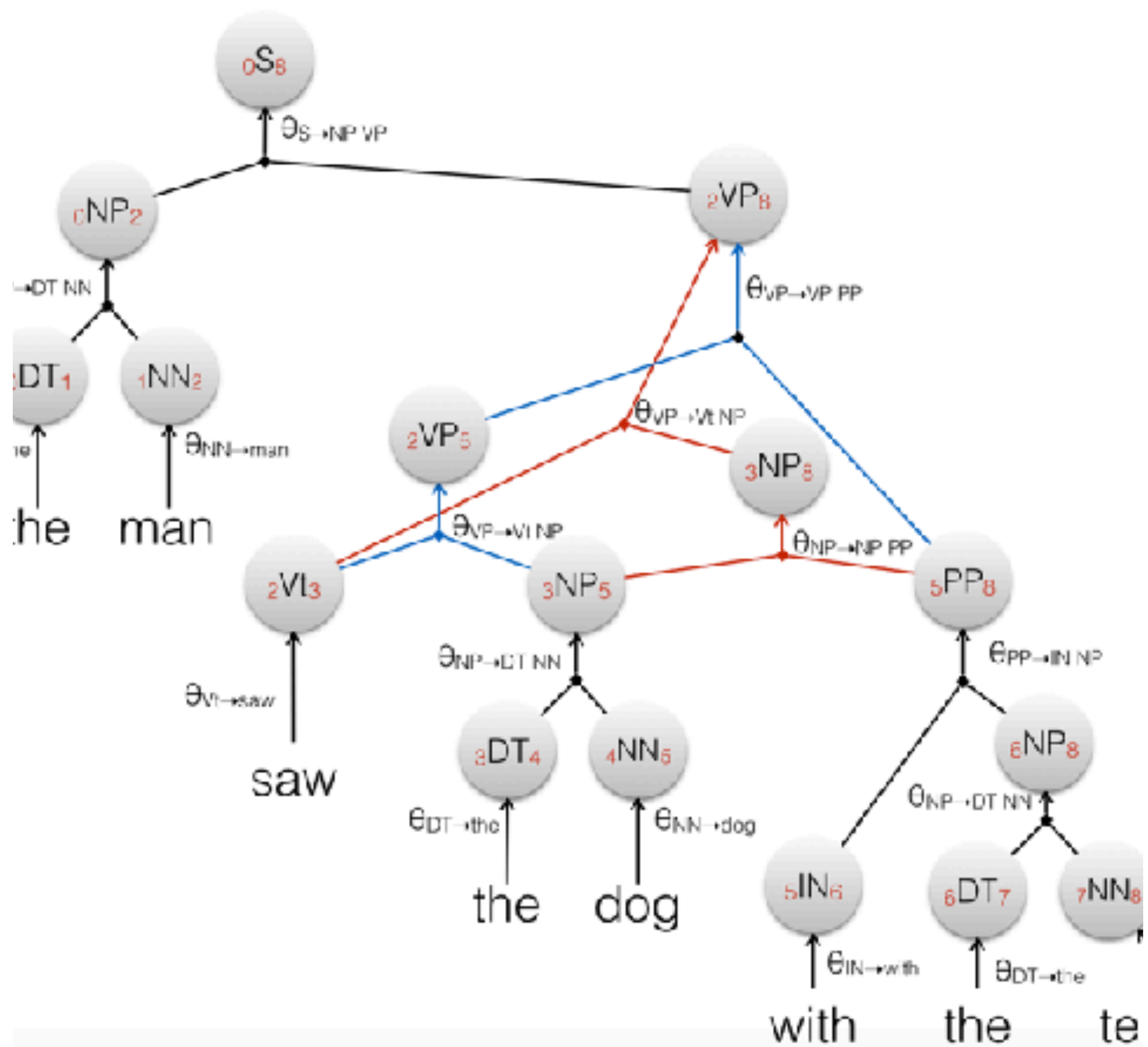
  $\theta_{S\rightarrow NP\ VP} \otimes I({}_0NP_2) \otimes I({}_2VP_8)$

- $I({}_0NP_2) =$

  $\theta_{NP\rightarrow DT\ NN} \otimes I({}_0DT_1) \otimes I({}_1NN_2)$

- $I({}_0DT_1) =$

  $\theta_{DT\rightarrow the} \otimes I(the)$



Inside example

- $I({}_0S_8) =$

  $\theta_{S \to NP\ VP} \otimes I({}_0NP_2) \otimes I({}_2VP_8)$

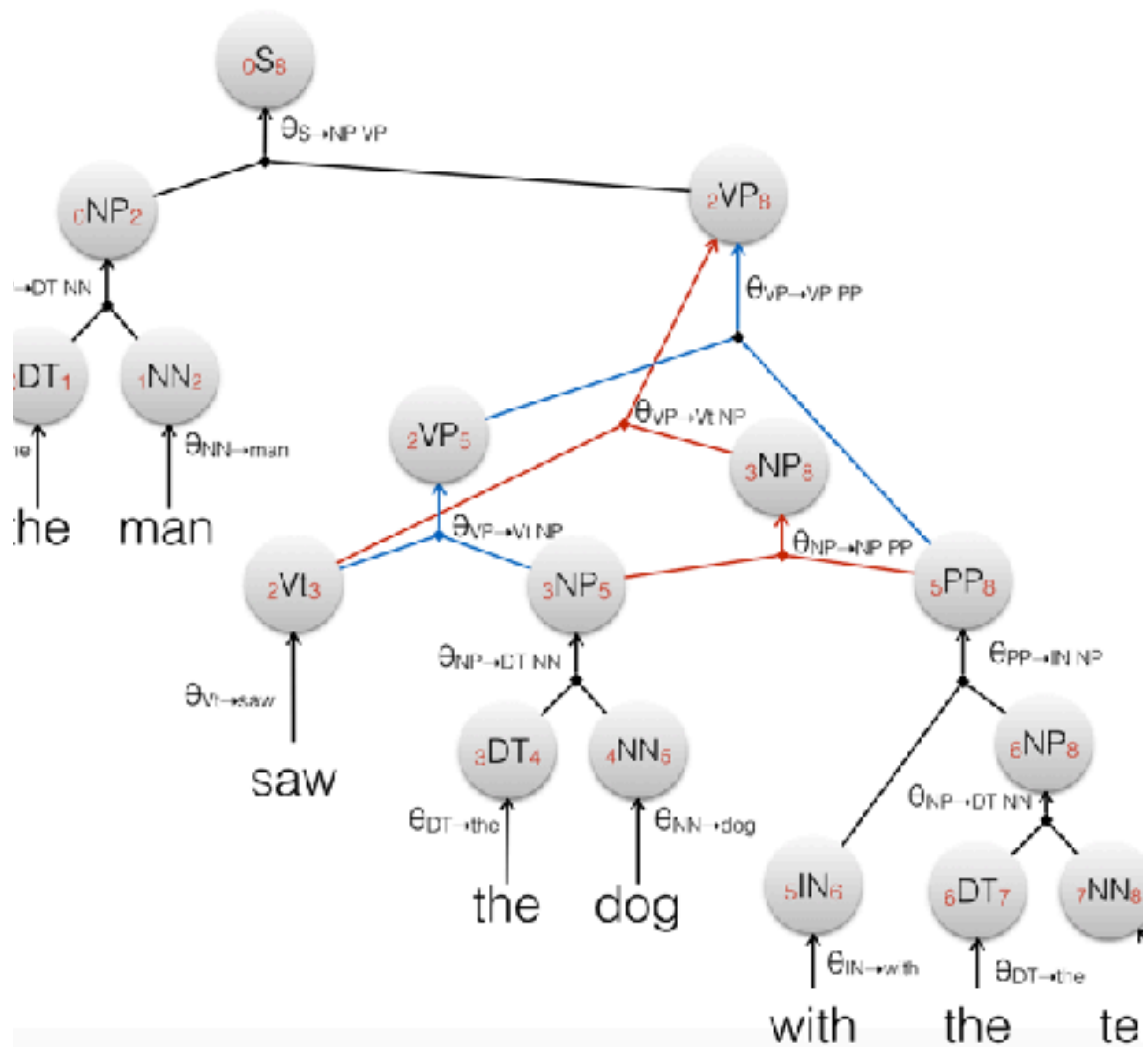
- $I({}_0NP_2) =$

  $\theta_{NP \to DT\ NN}$
  $\otimes I({}_0DT_1) \otimes I({}_1NN_2)$

- $I({}_0DT_1) =$

  $\theta_{DT \to the} \otimes I(the)$

- $I(the) = 1$



Complexity

$$X \to \alpha \bullet \beta \quad \text{where } |\alpha| > 1 \text{ and } |\beta| = 1$$

$$\Rightarrow X \to v(\alpha)\,\beta$$

$$v(\alpha) \to \alpha \quad \text{where } v(\alpha) \text{ turns } \alpha \text{ in a nonterminal}$$

Complexity

Every CFG can be binarised (max arity = 2)

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- In total we get up to 3 indices ranging from 0 .. n

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- In total we get up to 3 indices ranging from 0 .. n
- O(n^3) annotated rules

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