# Natural Language Models and Interfaces

## BSc Artificial Intelligence

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Institute for Logic, Language, and Computation

2019, week 5, lecture a

# NLMI

Trees and grammars

Context-free grammars

Probabilistic context-free grammars

# Modelling language so far

Bag-of-word models (or unigram LMs)

- ignore word order entirely

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HMM models

- capture a shortened fixed-length history
- by abstracting away from word form through word classes

# Long-distance dependencies

The form of one word often depends on (agrees with) another even when arbitrarily long material intervenes

- Sam sleeps soundly
- Dogs sleep soundly

Adapted from T. Deoskar

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- Sam, who is my cousin, sleeps soundly
- Dogs often play around my house and then sleep soundly

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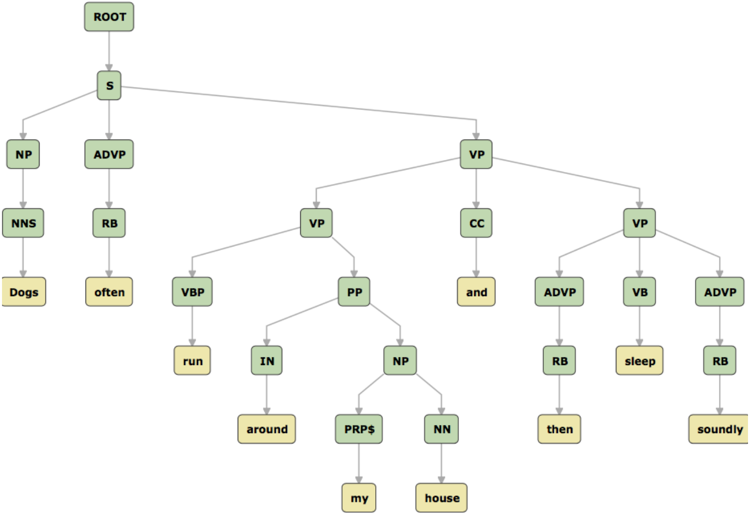
We want models that can capture these dependencies

- and are less sensitive to distance in linear order

Adapted from T. Deoskar

# What if we organise words and phrases in a tree?




CoreNLP

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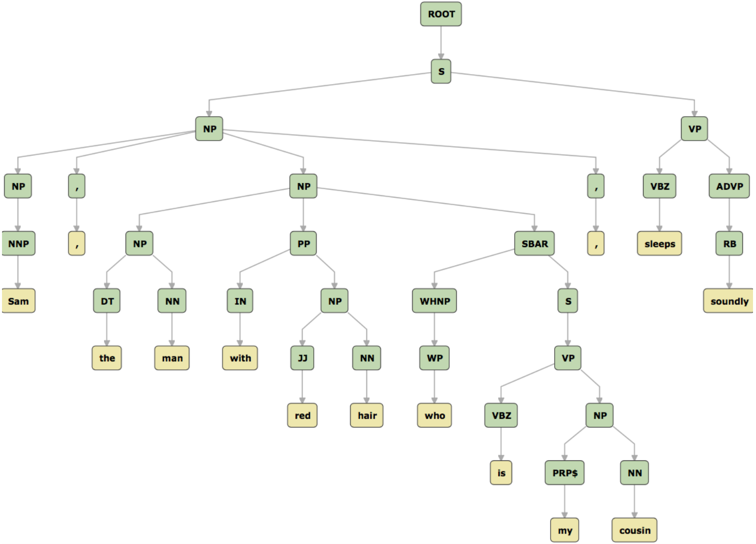


CoreNLP

# Phrases

Words are organised into groups (phrases) which function as a unit

- POS categories indicate which words are substitutable.
  e.g., substituting adjectives
  - I saw a red cat
  - I saw a former cat
  - I saw a sleepy cat
- Phrasal categories indicate which phrases are substitutable
  e.g., substituting noun phrase
  - Dogs sleep soundly
  - My next-door neighbours sleep soundly
  - Green ideas sleep soundly

Phrasal categories: noun phrase (NP), verb phrase (VP), prepositional phrase (PP), etc.

Adapted from T. Deoskar

# Heads and Phrases

The class that a word belongs to is closely linked to the name of the phrase it customarily appears in.

- In a X-phrase (e.g. NP), the key occurrence of X (e.g. N) is called the head.

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English NPs are commonly of the form

- (Det) Adj* **Noun** (PP — RelClause)*
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VPs are commonly of the form

- (Aux) Adv* **Verb** Arg* Adjunct*
  **VP**: usually eats pasta for dinner ; **head : eat**

Adapted from T. Deoskar

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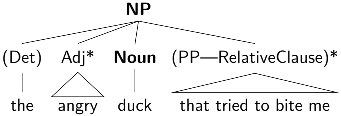


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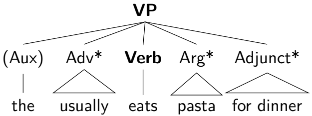

# Theories of Syntax

A theory of syntax should explain which sentences are well-formed (grammatical) and which are not

- *well-formed* is distinct from *meaningful*.
- Example from Chomsky
  Colorless green ideas sleep furiously

Adapted from T. Deoskar

# Theories of Syntax

A theory of syntax should explain which sentences are well-formed (grammatical) and which are not

- *well-formed* is distinct from *meaningful*.
- Example from Chomsky
  Colorless green ideas sleep furiously
- However, the reason we care about syntax is mainly for interpreting meaning

Adapted from T. Deoskar

# Desirable properties of a grammar

Chomsky specified two properties that make a grammar "interesting and satisfying"

- It should be a finite specification of the strings of the language, rather than a list of its sentences.
- It should be revealing, in allowing strings to be associated with meaning (semantics) in a systematic way.

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We can add another desirable property

- It should capture structural and distributional properties of the language
  e.g. where heads of phrases are located
  e.g. how a sentence transforms into a question
  e.g. which phrases can move around the sentence

Adapted from T. Deoskar

# Desirable properties of a grammar

- Context-free grammars (CFGs) provide a pretty good approximation
- Some features of NLs are more easily captured using *mildly context-sensitive* grammars
  - Combinatory Categorial Grammar (CCG)
  - Lexicalised Tree Adjoining Grammar (LTAG)

Adapted from T. Deoskar

# A small fragment of English

Let's say we want to capture in a grammar the structural and distributional properties that give rise to sentences like this:

| | |
|---|---|
| A duck walked in the park. | NP,V,PP |
| The man walked with a duck. | NP,V,PP |
| You made a duck. | Pro,V,NP |
| You made her duck. | ? Pro,V,NP |
| A man with a telescope saw you. | NP,PP,V,Pro |
| A man saw you with a telescope. | NP,V,Pro,PP |
| You saw a man with a telescope. | Pro,V,NP,PP |

- write lexical rules that generate the words appearing in them
- write grammatical rules that generate these phrase structures

Adapted from T. Deoskar

# Grammar for the small fragment of English

Grammar G1 generates the sentences on the previous slide:

| **Grammatical rules** | **Lexical rules** |
|---|---|
| S $\rightarrow$ NP VP | Det $\rightarrow$ a $\mid$ the $\mid$ her (determiners) |
| NP $\rightarrow$ Det N | N $\rightarrow$ man $\mid$ park $\mid$ duck $\mid$ telescope (nouns) |
| NP $\rightarrow$ Det N PP | Pro $\rightarrow$ you (pronoun) |
| NP $\rightarrow$ Pro | V $\rightarrow$ saw $\mid$ walked $\mid$ made (verbs) |
| VP $\rightarrow$ V NP PP | Prep $\rightarrow$ in $\mid$ with $\mid$ for (prepositions) |
| VP $\rightarrow$ V NP | |
| VP $\rightarrow$ V | |
| PP $\rightarrow$ Prep NP | |

Does G1 produce a finite or an infinite number of sentences?

Adapted from T. Deoskar

# Recursion

Recursion in a grammar makes it possible to generate an infinite number of sentences

- Direct recursion: a non-terminal on the LHS of a rule also appears on its RHS
  VP $\rightarrow$ VP Conj VP
  Conj $\rightarrow$ and — or
- Indirect recursion: some non-terminal can be expanded (in several steps) to a sequence of symbols containing that non-terminal
  NP $\rightarrow$ Det N PP
  PP $\rightarrow$ Prep NP

# NLMI

Trees and grammars

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# Context-Free Grammar

A rewriting system with two types of symbols

- Terminals (or *constants*): words
- Nonterminals (or *variables*): phrasal categories
  e.g. S, NP, VP
  with S being the start symbol

Rules of the form X $\rightarrow \beta$
where $\beta$ is any string of nonterminals and terminals
indicate that X can be replaced by $\beta$ anywhere where X occurs

# CFG example

| | |
|---|---|
| S → NP VP | **(Sentences)** |
| NP → D N \| Pro \| PropN | **(Noun phrases)** |
| D → PosPro \| Art \| NP 's | **(Determiners)** |
| VP → Vi \| Vt NP \| Vp NP VP | **(Verb phrases)** |
| Pro → i \| we \| you \| he \| she \| him \| her | **(Pronouns)** |
| PosPro → my \| our \| your \| his \| her | **(Possessive pronouns)** |
| PropN → Robin \| Jo | **(Proper nouns)** |
| Art → a \| an \| the | **(Articles)** |
| N → man \| duck \| saw \| park \| telescope | **(Nouns)** |
| Vi → sleep \| run \| duck | **(Intransitive verbs)** |
| Vt → eat \| break \| see \| saw | **(Transitive verbs)** |
| Vp → see \| saw \| heard | **(Verbs with NP VP args)** |

# CFGs formally

Let $\Sigma$ be a finite set of terminal symbols (aka words)
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Let $\mathcal{R} \subseteq \mathcal{V} \times (\Sigma \cup \mathcal{V})^*$ be a finite set of rules of the form
 $\mathsf{X} \to \beta$ where $\mathsf{X} \in \mathcal{V}$ and $\beta \in (\Sigma \cup \mathcal{V})^*$

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A CFG is the tuple $\mathfrak{G} = \langle \Sigma, \mathcal{V}, \mathsf{S}, \mathcal{R} \rangle$

# CFG terminology

The number of symbols on the RHS is the arity of the rule

- unary: $\mathsf{X} \rightarrow \mathsf{Y}$
- binary: $\mathsf{X} \rightarrow \mathsf{Y}\,\mathsf{Z}$
- $n$-ary: $\mathsf{X} \rightarrow \mathsf{X}_1 \cdots \mathsf{X}_n$
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- phrase rules? $O(|\mathcal{V}| \times |\Sigma \cup \mathcal{V}|^a)$

# Derivation

We can use CFGs to derive strings

A derivation is a sequence of strings

- we start from the string $\langle \mathsf{S} \rangle$
- and at each step we rewrite the leftmost nonterminal X by application of a rule $\mathsf{X} \rightarrow \beta$
- until only terminals remain
  which we denote $\mathsf{S} \overset{*}{\Rightarrow} \mathsf{x}_1 \cdots \mathsf{x}_n$

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2. $\langle \mathsf{NP}\ \mathsf{VP} \rangle$

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1. $\langle \mathsf{S} \rangle$
2. $\langle \mathsf{NP\ VP} \rangle$
3. $\langle \mathsf{D\ N\ VP} \rangle$

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Example

1. $\langle \mathsf{S} \rangle$
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Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
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5. $\langle$the dog VP$\rangle$
6. $\langle$the dog V$\rangle$

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2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
4. $\langle$the N VP$\rangle$
5. $\langle$the dog VP$\rangle$
6. $\langle$the dog V$\rangle$
7. $\langle$the dog barks$\rangle$

# Derivation - examples

Example

1. $\langle \mathrm{S} \rangle$

Example

# Derivation - examples

Example

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$

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Example

# Derivation - examples

Example

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5. $\langle$the dog VP$\rangle$
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Example

# Derivation - examples

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
4. $\langle$the N VP$\rangle$
5. $\langle$the dog VP$\rangle$
6. $\langle$the dog V$\rangle$
7. $\langle$the dog runs$\rangle$

Example

# Derivation - examples

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
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Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$

# Derivation - examples

Example

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2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
4. $\langle$the N VP$\rangle$
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6. $\langle$the dog V$\rangle$
7. $\langle$the dog runs$\rangle$

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$N VP$\rangle$

# Derivation - examples

Example

1. $\langle S \rangle$
2. $\langle NP\ VP \rangle$
3. $\langle D\ N\ VP \rangle$
4. $\langle \text{the}\ N\ VP \rangle$
5. $\langle \text{the dog}\ VP \rangle$
6. $\langle \text{the dog}\ V \rangle$
7. $\langle \text{the dog runs} \rangle$

Example

1. $\langle S \rangle$
2. $\langle NP\ VP \rangle$
3. $\langle N\ VP \rangle$
4. $\langle \text{cats}\ VP \rangle$

# Derivation - examples

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
4. $\langle$the N VP$\rangle$
5. $\langle$the dog VP$\rangle$
6. $\langle$the dog V$\rangle$
7. $\langle$the dog runs$\rangle$

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$N VP$\rangle$
4. $\langle$cats VP$\rangle$
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# Derivation - examples

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2. $\langle$NP VP$\rangle$
3. $\langle$N VP$\rangle$
4. $\langle$cats VP$\rangle$
5. $\langle$cats V$\rangle$
6. $\langle$cats run$\rangle$

# Derivation - more examples

1. $\langle \mathsf{S} \rangle$

# Derivation - more examples

1. $\langle \mathrm{S} \rangle$
2. $\langle \mathrm{NP\ VP} \rangle$

# Derivation - more examples

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$NP CC NP VP$\rangle$

# Derivation - more examples

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$
3. $\langle \text{NP CC NP VP} \rangle$
4. $\langle \text{N CC NP VP} \rangle$

# Derivation - more examples

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$
3. $\langle \text{NP CC NP VP} \rangle$
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# Derivation - more examples

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
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4. $\langle$N CC NP VP$\rangle$
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8. $\langle$cats and dogs VP$\rangle$

# Derivation - more examples

1. $\langle$S$\rangle$
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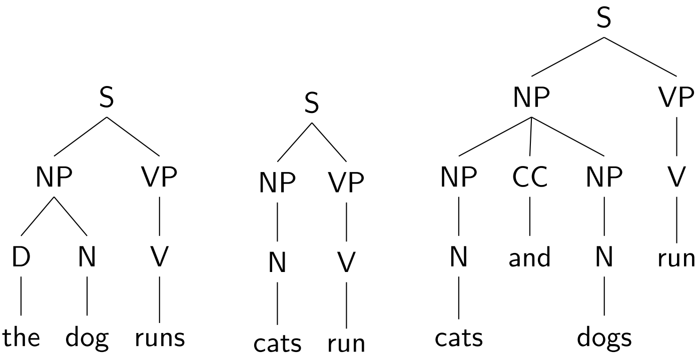
7. $\langle$cats and N VP$\rangle$
8. $\langle$cats and dogs VP$\rangle$
9. $\langle$cats and dogs V$\rangle$

# Derivation - more examples

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$NP CC NP VP$\rangle$
4. $\langle$N CC NP VP$\rangle$
5. $\langle$cats CC NP VP$\rangle$
6. $\langle$cats and NP VP$\rangle$
7. $\langle$cats and N VP$\rangle$
8. $\langle$cats and dogs VP$\rangle$
9. $\langle$cats and dogs V$\rangle$
10. $\langle$cats and dogs run$\rangle$

# Parse trees

Parse trees compactly represent derivations

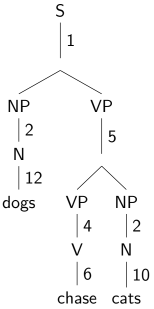


# Derivation: a sequence of rule applications

A derivation can be seen as a sequence of rule applications $\langle r_1, \ldots, r_m \rangle$

- starts from S
- and after $m$ steps yields a string $\text{yield}(r_1^m) = x_1^n$
- the sequence can be read off of a tree by a depth-first traversal

# Derivations as trees




1. S → NP VP
2. NP → N
3. NP → D N
4. VP → V
5. VP → VP NP
6. V → chase
7. V → chases
8. D → the
9. N → cat
10. N → cats
11. N → dog
12. N → dogs

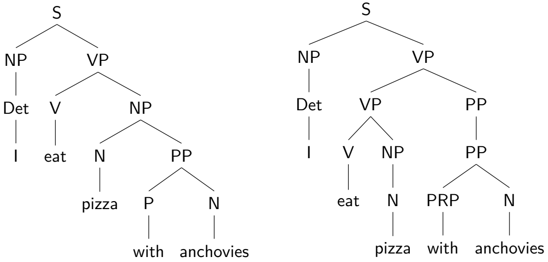
Sequence of rule applications (depth-first traversal)

$\langle r_1 = 1, r_2 = 2, r_3 = 12, r_4 = 5, r_5 = 4, r_6 = 6, r_7 = 2, r_8 = 10 \rangle$

The sentence is the yield of the derivation

# Structural ambiguity

Different structure leads to different interpretation



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Different structure leads to different interpretation


[S [NP [Det I]] [VP [V eat] [NP [N pizza] [PP [P with] [N anchovies]]]]]

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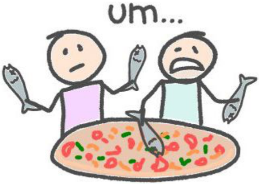

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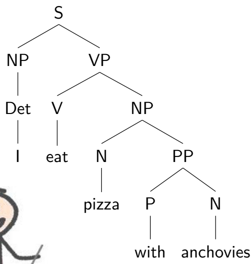


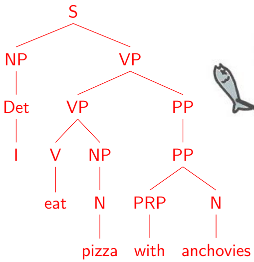


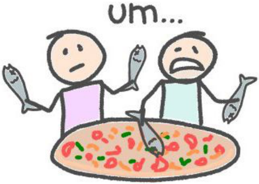


How should we deal with this?

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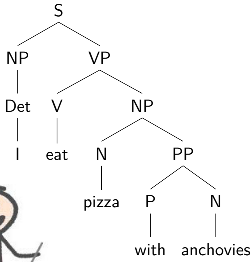


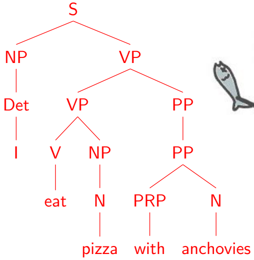


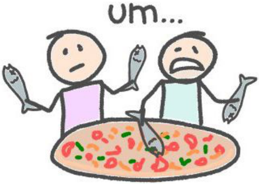


How should we deal with this? Probabilities!

# NLMI

Trees and grammars

Context-free grammars

Probabilistic context-free grammars

# Probability distributions over derivations

We define a random derivation $D$

- a sequence $\langle R_1, \ldots, R_m \rangle$ of random rule applications
- where $R$ is a random variable indexing rules of the grammar

The probability over a sequence of $m$ rules can be written

$$P_{D|M}(r_1^m|m) = \underbrace{\prod_{i=1}^{m} P_{R_i|R_{<i}}(r_i|r_{<i})}_{\text{chain rule}}$$

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$$\approx \underbrace{\prod_{i=1}^{m} P_R(r_i)}_{\text{independence assumption}}$$

# Conditional independence

A rule rewrites a LHS nonterminal symbol into a RHS string

- A rule $R : v \to \beta$ corresponds to a random pair $(\mathrm{LHS}, \text{RHS})$
- $\mathrm{LHS}$ corresponds to a random nonterminal symbol $v \in \mathcal{V}$
- $\mathrm{RHS}$ corresponds to a random sequence of terminals and nonterminals $\beta \in (\Sigma \cup \mathcal{V})^a$

Then we re-write the probability of a derivation as

$$P_{D|M}(r_1^m|m) = P_D(\langle\underbrace{(v_1, \beta_1)}_{r_1}, \ldots, \underbrace{(v_m, \beta_m)}_{r_m}\rangle|m)$$

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&= \prod_{i=1}^{m} P_R(r_i) = \prod_{i=1}^{m} P_R(v_i \to \beta_i) \\
&= \prod_{i=1}^{m} P_{\mathrm{RHS}|\mathrm{LHS}}(\beta_i | v_i)
\end{aligned}
$$

# Factorisation

We make $\mathrm{RHS}|\mathrm{LHS} = v \sim \mathrm{Cat}(\theta^{(v)})$

- $0 < \theta_{v \to \beta} < 1$
- $\sum_\beta \theta_{v \to \beta} = 1$

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- $P_R(v \to \beta) = P_{\mathrm{RHS}|\mathrm{LHS}}(\beta|v) = \theta_{v \to \beta}$ or $P_R(r) = \theta_r$
  e.g. $P_R(\mathsf{S} \to \mathsf{NP\ VP}) = P_{\mathrm{RHS}|\mathsf{LHS}}(\mathsf{NP\ VP}|\mathsf{S}) = \theta_{\mathsf{S} \to \mathsf{NP\ VP}}$

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How many parameters to represent $P_R$?

- One cpd per LHS, thus $O(|\mathcal{V}| \times |\Sigma \cup \mathcal{V}|^a)$

# Probability of a derivation

A simple product over $m$ terms

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A simple product over $m$ terms

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A simple product over $m$ terms

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P_{D|M}(r_1^m|m) &= \prod_{i=1}^{m} P_R(v_i \to \beta_i) \\
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\end{aligned}
$$

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# Probabilistic CFG Language Model

Generative story

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$$P_{SD}(x_1^n, r_1^m) = P_N(n) P_{M|N}(m|n) \underbrace{P_{D|M}(r_1^m) P_{S|DNM}(x_1^n|r_1^m)}_{P_{SD|NM}(x_1^n, r_1^m|n, m)}$$

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But note that (4) is deterministic

$$P_{S|DNM}(x_1^n|r_1^m) = \begin{cases} 1 & \text{if } \operatorname{yield}(r_1^m) = x_1^n \\ 0 & \text{otherwise} \end{cases}$$

# Probability of a sentence

Joint distribution

$$P_{SD}(x_1^n, r_1^m) = P_N(n)P_{M|N}(m|n)P_{D|M}(r_1^m)P_{S|DNM}(x_1^n|r_1^m)$$

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where $\mathfrak{G}(x_1^n)$ is the set of derivations whose yield is $x_1^n$

# Probability of a sentence

Typically $P_N$ and $P_{M|N}$ are ignored (assumed uniform), then

$$P_S(x_1^n) \propto \sum_{r_1^m \in \mathfrak{G}(x_1^n)} P_{D|M}(r_1^m)$$

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Challenge: to express $\mathfrak{G}(x_1^n)$ a task called parsing

# Maximum likelihood estimation

We have a treebank, that is, a corpus where

- a sentence $x_1^n$ is annotated with its CFG tree $r_1^m$

Our distributions $P_{\text{RHS}|\text{LHS}}$ are categorical

- $\text{RHS} \mid \text{LHS} = v \sim \text{Cat}(\theta^{(v)})$

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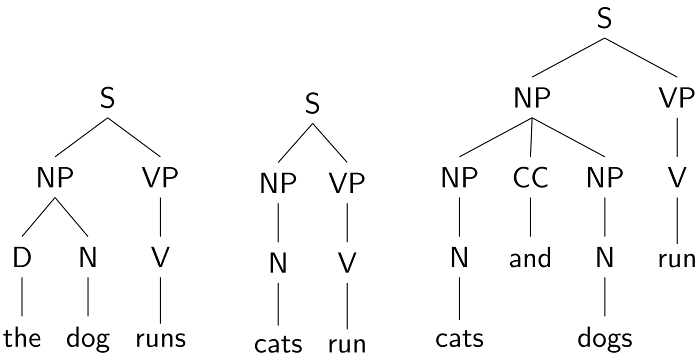
MLE solution?

$$\theta_{v \to \beta} = \frac{\text{count}_R(v \to \beta)}{\sum_{\beta'} \text{count}_R(v \to \beta')} = \frac{\text{count}_R(v \to \beta)}{\text{count}_{\text{LHS}}(v)}$$

# MLE - Example

Consider the treebank




- $\theta_{\text{NP}\to\text{N}}$
- $\theta_{\text{N}\to\text{dog}}$

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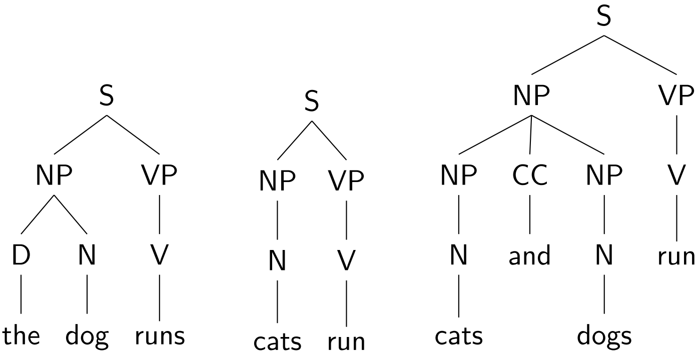


- $\theta_{\text{NP}\to\text{N}} = \frac{\text{count}(\text{NP}\to\text{N})}{\text{count}(\text{NP}\to *)}$
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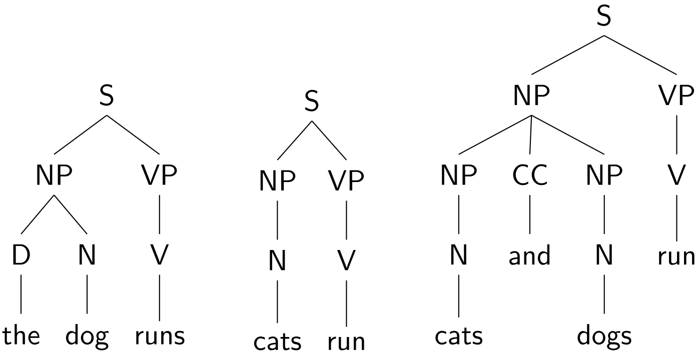


- $\theta_{\text{NP}\to\text{N}} = \frac{\text{count}(\text{NP}\to\text{N})}{\text{count}(\text{NP}\to *)} = \frac{3}{5}$
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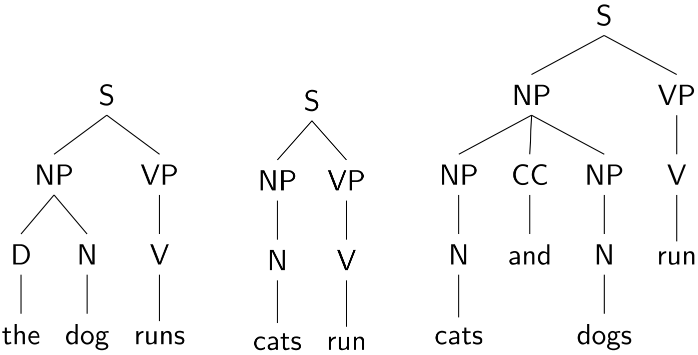


- $\theta_{\mathsf{NP}\to\mathsf{N}} = \frac{\text{count}(\mathsf{NP}\to\mathsf{N})}{\text{count}(\mathsf{NP}\to *)} = \frac{3}{5}$
- $\theta_{\mathsf{N}\to\mathsf{dog}} = \frac{\text{count}(\mathsf{N}\to\mathsf{dog})}{\text{count}(\mathsf{N}\to *)}$

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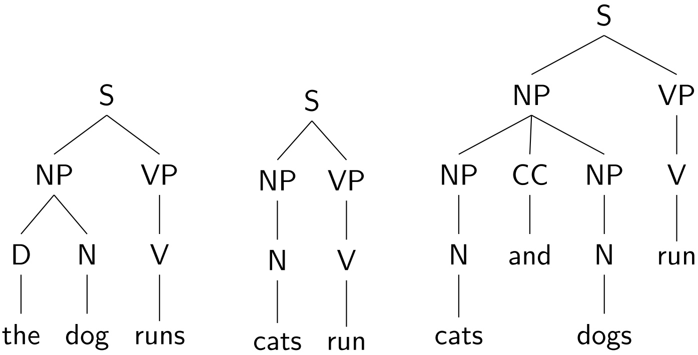


- $\theta_{\text{NP}\rightarrow\text{N}} = \frac{\text{count}(\text{NP}\rightarrow\text{N})}{\text{count}(\text{NP}\rightarrow *)} = \frac{3}{5}$
- $\theta_{\text{N}\rightarrow\text{dog}} = \frac{\text{count}(\text{N}\rightarrow\text{dog})}{\text{count}(\text{N}\rightarrow *)} = \frac{1}{4}$

# References I